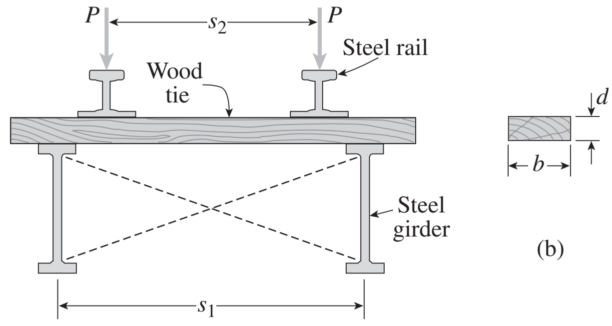
Design of Beams

Problem 5.6-1 The cross section of a narrow-gage railway bridge is shown in part (a) of the figure. The bridge is constructed with longitudinal steel girders that support the wood cross ties. The girders are restrained against lateral buckling by diagonal bracing, as indicated by the dashed lines.

The spacing of the girders is $s_1 = 50$ in. and the spacing of the rails is $s_2 = 30$ in. The load transmitted by each rail to a single tie is $P = 1500$ lb. The cross section of a tie, shown in part (b) of the figure, has width $b = 5.0$ in. and depth d.

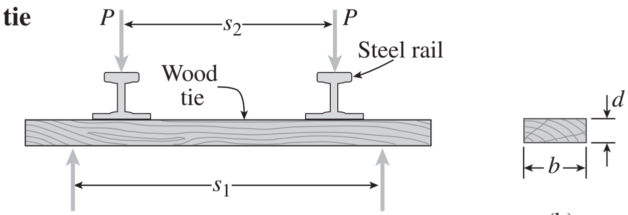
Determine the minimum value of d based upon an allowable bending stress of 1125 psi in the wood tie. (Disregard the weight of the tie itself.)



(b)

(a)

Solution 5.6-1 **Railway cross tie**



(b)

(a)

$s_1 = 50$ in. $\quad b = 5.0$ in. $\quad s_2 = 30$ in.

d = depth of tie $\quad P = 1500$ lb $\quad \sigma_{allow} = 1125$ psi

$$M_{max} = \frac{P(s_1 - s_2)}{2} = 15{,}000 \text{ lb-in.}$$

$$S = \frac{bd^2}{6} = \frac{1}{6}(50 \text{ in.})(d^2) = \frac{5d^2}{6} \qquad d = \text{inches}$$

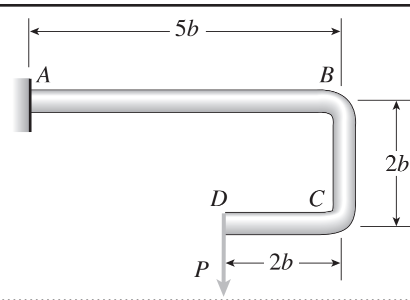
$$M_{max} = \sigma_{allow} S \qquad 15{,}000 = (1125)\left(\frac{5d^2}{6}\right)$$

Solving, $d_2 = 16.0$ in. $\quad d_{min} = 4.0$ in. ←

Note: Symbolic solution: $d^2 = \dfrac{3P(s_1 - s_2)}{b\sigma_{allow}}$

Problem 5.6-2 A fiberglass bracket *ABCD* of solid circular cross section has the shape and dimensions shown in the figure. A vertical load $P = 36$ N acts at the free end D.

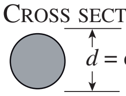
Determine the minimum permissible diameter d_{min} of the bracket if the allowable bending stress in the material is 30 MPa and $b = 35$ mm. (Disregard the weight of the bracket itself.)



Solution 5.6-2 **Fiberglass bracket**

DATA $\quad P = 36$ N $\quad \sigma_{allow} = 30$ MPa $\quad b = 35$ mm

CROSS SECTION



$$I = \frac{\pi d^4}{64}$$

MAXIMUM BENDING MOMENT $\quad M_{max} = P(3b)$

MAXIMUM BENDING STRESS

$$\sigma_{max} = \frac{M_{max}\, c}{I} \qquad c = \frac{d}{2} \qquad \sigma_{allow} = \frac{3Pbd}{2I} = \frac{96\, Pb}{\pi d^3}$$

MINIMUM DIAMETER

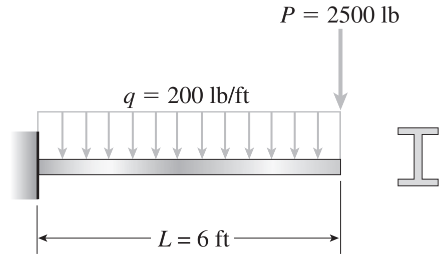
$$d^3 = \frac{96Pb}{\pi\sigma_{allow}} = \frac{(96)(36\text{ N})(35\text{ mm})}{\pi(30\text{ MPa})}$$

$$= 1{,}283.4 \text{ mm}^3$$

$d_{min} = 10.9$ mm ←

Problem 5.6-3 A cantilever beam of length $L = 6$ ft supports a uniform load of intensity $q = 200$ lb/ft and a concentrated load $P = 2500$ lb (see figure).

Calculate the required section modulus S if $\sigma_{allow} = 15{,}000$ psi. Then select a suitable wide-flange beam (W shape) from Table E-1, Appendix E, and recalculate S taking into account the weight of the beam. Select a new beam size if necessary.



Solution 5.6-3 **Cantilever beam**

$P = 2500$ lb $\quad q = 200$ lb/ft $\quad L = 6$ ft

$\sigma_{allow} = 15{,}000$ psi

Required section modulus

$$M_{max} = PL + \frac{qL^2}{2} = 15{,}000 \text{ lb-ft} + 3{,}600 \text{ lb-ft}$$

$$= 18{,}600 \text{ lb-ft} = 223{,}200 \text{ lb-in.}$$

$$S = \frac{M_{max}}{\sigma_{allow}} = \frac{223{,}200 \text{ lb-in.}}{15{,}000 \text{ psi}} = 14.88 \text{ in.}^3$$

Trial section W 8 × 21

$S = 18.2 \text{ in.}^3 \quad q_0 = 21$ lb/ft

$$M_0 = \frac{q_0 L^2}{2} = 378 \text{ lb-ft} = 4536 \text{ lb-in.}$$

$$M_{max} = 223{,}200 + 4{,}536 = 227{,}700 \text{ lb-in.}$$

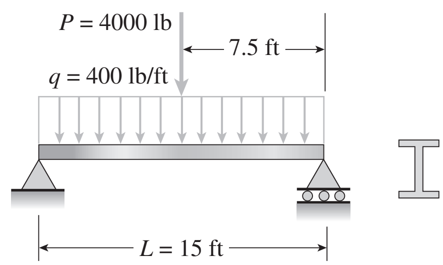
$$\text{Required } S = \frac{M_{max}}{\sigma_{allow}} = \frac{227{,}700 \text{ lb-in.}}{15{,}000 \text{ psi}} = 15.2 \text{ in.}^3$$

$15.2 \text{ in.}^3 < 18.2 \text{ in.}^3 \quad \therefore$ Beam is satisfactory.

Use W 8 × 21 ←

Problem 5.6-4 A simple beam of length $L = 15$ ft carries a uniform load of intensity $q = 400$ lb/ft and a concentrated load $P = 4000$ lb (see figure).

Assuming $\sigma_{allow} = 16{,}000$ psi, calculate the required section modulus S. Then select an 8-inch wide-flange beam (W shape) from Table E-1, Appendix E, and recalculate S taking into account the weight of the beam. Select a new 8-inch beam if necessary.



Solution 5.6-4 **Simple beam**

$P = 4000$ lb $\quad q = 400$ lb/ft $\quad L = 15$ ft

$\sigma_{allow} = 16{,}000$ psi $\quad$ use an 8-inch W shape

Required section modulus

$$M_{max} = \frac{PL}{4} + \frac{qL^2}{8} = 15{,}000 \text{ lb-ft} + 11{,}250 \text{ lb-ft}$$

$$= 26{,}250 \text{ lb-ft} = 315{,}000 \text{ lb-in.}$$

$$S = \frac{M_{max}}{\sigma_{allow}} = \frac{315{,}000 \text{ lb-in.}}{16{,}000 \text{ psi}} = 19.69 \text{ in.}^3$$

Trial section W 8 × 28

$S = 24.3 \text{ in.}^3 \quad q_0 = 28$ lb/ft

$$M_0 = \frac{q_0 L^2}{8} = 787.5 \text{ lb-ft} = 9450 \text{ lb-in.}$$

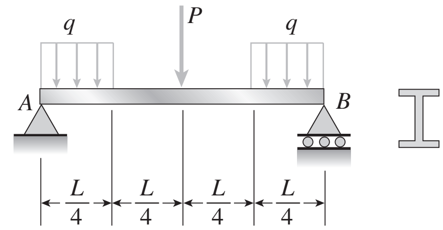
$$M_{max} = 315{,}000 + 9{,}450 = 324{,}450 \text{ lb-in.}$$

$$\text{Required } S = \frac{M_{max}}{\sigma_{allow}} = \frac{324{,}450 \text{ lb-in.}}{16{,}000 \text{ psi}} = 20.3 \text{ in.}^3$$

$20.3 \text{ in.}^3 < 24.3 \text{ in.}^3 \quad \therefore$ Beam is satisfactory.

Use W 8 × 28 ←

Problem 5.6-5 A simple beam AB is loaded as shown in the figure on the next page. Calculate the required section modulus S if $\sigma_{\text{allow}} = 15{,}000$ psi, $L = 24$ ft, $P = 2000$ lb, and $q = 400$ lb/ft. Then select a suitable I-beam (S shape) from Table E-2, Appendix E, and recalculate S taking into account the weight of the beam. Select a new beam size if necessary.



Solution 5.6-5 Simple beam

$P = 2000$ lb $q = 400$ lb/ft $L = 24$ ft

$\sigma_{\text{allow}} = 15{,}000$ psi

REQUIRED SECTION MODULUS

$$M_{\max} = \frac{PL}{4} + \frac{qL^2}{32} = 12{,}000 \text{ lb-ft} + 7{,}200 \text{ lb-ft}$$

$$= 19{,}200 \text{ lb-ft} = 230{,}400 \text{ lb-in.}$$

$$S = \frac{M_{\max}}{\sigma_{\text{allow}}} = \frac{230{,}400 \text{ lb-in.}}{15{,}000 \text{ psi}} = 15.36 \text{ in.}^3$$

TRIAL SECTION S 10 × 25.4

$S = 24.7 \text{ in.}^3$ $q_0 = 25.4$ lb/ft

$$M_0 = \frac{q_0 L^2}{8} = 1829 \text{ lb-ft} = 21{,}950 \text{ lb-in.}$$

$$M_{\max} = 230{,}400 + 21{,}950 = 252{,}300 \text{ lb-in.}$$

$$\text{Required } S = \frac{M_{\max}}{\sigma_{\text{allow}}} = \frac{252{,}300 \text{ lb-in.}}{15{,}000 \text{ psi}} = 16.8 \text{ in.}^3$$

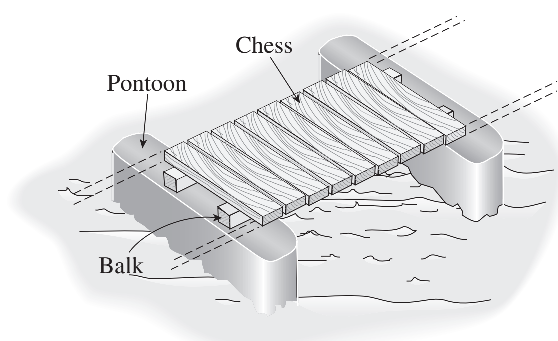
$16.8 \text{ in.}^3 < 24.7 \text{ in.}^3$ ∴ Beam is satisfactory.

Use S 10 × 25.4 ←

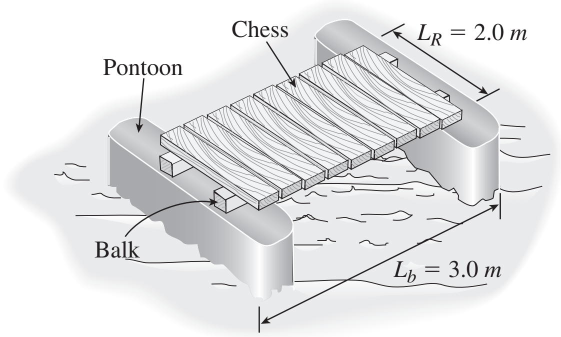
Problem 5.6-6 A pontoon bridge (see figure) is constructed of two longitudinal wood beams, known as *balks*, that span between adjacent pontoons and support the transverse floor beams, which are called *chesses*.

For purposes of design, assume that a uniform floor load of 8.0 kPa acts over the chesses. (This load includes an allowance for the weights of the chesses and balks.) Also, assume that the chesses are 2.0 m long and that the balks are simply supported with a span of 3.0 m. The allowable bending stress in the wood is 16 MPa.

If the balks have a square cross section, what is their minimum required width $b_{\min}$?



Solution 5.6-6 Pontoon bridge

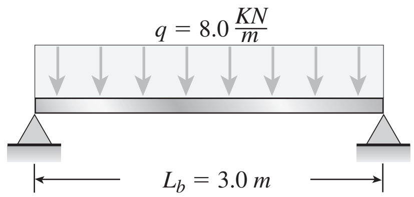


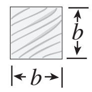
FLOOR LOAD: $w = 8.0$ kPa

ALLOWABLE STRESS: $\sigma_{\text{allow}} = 16$ MPa

L_c = length of chesses $= 2.0$ m L_b = length of balks $= 3.0$ m

LOADING DIAGRAM FOR ONE BALK





W = total load

$= wL_bL_c$

$$q = \frac{W}{2L_b} = \frac{wL_c}{2}$$

$$= \frac{(8.0 \text{ kPa})(2.0 \text{ m})}{2}$$

$= 8.0$ kN/m

Section modulus $S = \dfrac{b^3}{6}$

$$M_{\max} = \frac{qL_b^2}{8} = \frac{(8.0 \text{ kN/m})(3.0 \text{ m})^2}{8} = 9{,}000 \text{ N} \cdot \text{m}$$

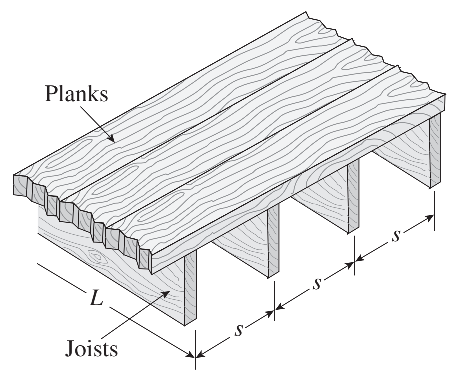
$$S = \frac{M_{\max}}{\sigma_{\text{allow}}} = \frac{9{,}000 \text{ N} \cdot \text{m}}{16 \text{ MPa}} = 562.5 \times 10^{-6} \text{ m}^3$$

$$\therefore \frac{b^3}{6} = 562.5 \times 10^{-6} \text{ m}^3 \quad \text{and} \quad b^3 = 3375 \times 10^{-6} \text{ m}^3$$

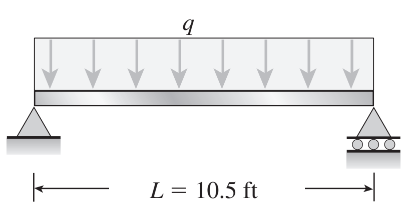
Solving, $b_{\min} = 0.150 \text{ m} = 150 \text{ mm}$ ←

Problem 5.6-7 A floor system in a small building consists of wood planks supported by 2 in. (nominal width) joists spaced at distance s, measured from center to center (see figure). The span length L of each joist is 10.5 ft, the spacing s of the joists is 16 in., and the allowable bending stress in the wood is 1350 psi. The uniform floor load is 120 lb/ft^2, which includes an allowance for the weight of the floor system itself.

Calculate the required section modulus S for the joists, and then select a suitable joist size (surfaced lumber) from Appendix F, assuming that each joist may be represented as a simple beam carrying a uniform load.



Solution 5.6-7 **Floor joists**



$\sigma_{\text{allow}} = 1350$ psi
$L = 10.5$ ft $= 126$ in.
w = floor load = 120 lb/ft^2 = 0.8333 lb/in.^{2}
s = spacing of joists = 16 in.
$q = ws = 13.333$ lb/in.

$$M_{\max} = \frac{qL^2}{8} = \frac{1}{8}(13.333 \text{ lb/in.})(126 \text{ in.})^2 = 26{,}460 \text{ lb-in.}$$

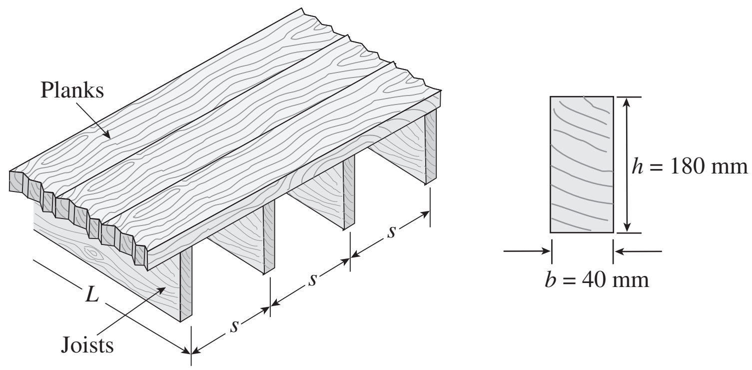
$$\text{Required } S = \frac{M_{\max}}{\sigma_{\text{allow}}} = \frac{26{,}460 \text{ lb/in.}}{1350 \text{ psi}} = 19.6 \text{ in.}^3$$ ←

From Appendix F: Select 2 × 10 in. joists ←

Problem 5.6-8 The wood joists supporting a plank floor (see figure) are 40 mm × 180 mm in cross section (actual dimensions) and have a span length $L = 4.0$ m. The floor load is 3.6 kPa, which includes the weight of the joists and the floor.

Calculate the maximum permissible spacing s of the joists if the allowable bending stress is 15 MPa. (Assume that each joist may be represented as a simple beam carrying a uniform load.)

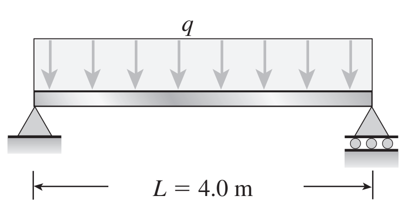
Solution 5.6-8 Spacing of floor joists



$L = 4.0$ m

w = floor load = 3.6 kPa $\quad \sigma_{allow} = 15$ MPa

s = spacing of joists



$$q = ws$$

$$S = \frac{bh^2}{6}$$

$$M_{max} = \frac{qL^2}{8} = \frac{wsL^2}{8}$$

$$S = \frac{M_{max}}{\sigma_{allow}} = \frac{wsL^2}{8\sigma_{allow}} = \frac{bh^2}{6}$$

Spacing of joists $\quad s_{max} = \dfrac{4\,bh^2\sigma_{allow}}{3wL^2}$ ←

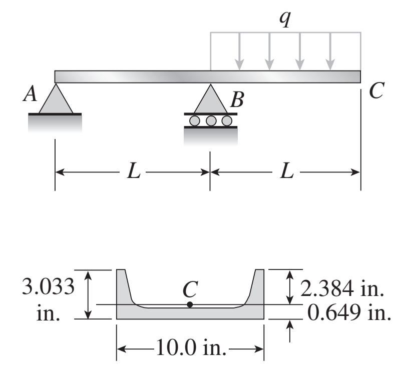
Substitute numerical values:

$$s_{max} = \frac{4(40 \text{ mm})(180 \text{ mm})^2(15 \text{ MPa})}{3(3.6 \text{ kPa})\ (4.0 \text{ m})^2}$$

$$= 0.450 \text{ m} = 450 \text{ mm} \quad \leftarrow$$

Problem 5.6-9 A beam *ABC* with an overhang from *B* to *C* is constructed of a C 10 × 30 channel section (see figure). The beam supports its own weight (30 lb/ft) plus a uniform load of intensity *q* acting on the overhang. The allowable stresses in tension and compression are 18 ksi and 12 ksi, respectively.

Determine the allowable uniform load q_{allow} if the distance *L* equals 3.0 ft.



Solution 5.6-9 **Beam with an overhang**

DATA

C 10 × 30 channel section

$c_1 = 2.384$ in. $c_2 = 0.649$ in.

$I = 3.94$ in.^{4} (from Table E-3)

q_0 = weight of beam ABC = 30 lb/ft = 2.5 lb/in.

q = load on overhang

L = length of overhang = 3.0 ft = 36 in.

ALLOWABLE STRESSES

$\sigma_t = 18$ ksi $\sigma_c = 12$ ksi

MAXIMUM BENDING MOMENT

M_{max} occurs at support B. $M_{max} = \dfrac{(q + q_0)L^2}{2}$

Tension on top; compression on bottom.

ALLOWABLE BENDING MOMENT BASED UPON TENSION

$$M_t = \frac{\sigma_t I}{c_1} = \frac{(18\text{ ksi})(3.94\text{ in.}^4)}{2.384\text{ in.}} = 29{,}750\text{ lb-in.}$$

ALLOWABLE BENDING MOMENT BASED UPON COMPRESSION

$$M_c = \frac{\sigma_c I}{c_2} = \frac{(12\text{ ksi})(3.94\text{ in.}^4)}{0.649\text{ in.}} = 72{,}850\text{ lb-in.}$$

ALLOWABLE BENDING MOMENT

Tension governs. $M_{allow} = 29{,}750$ lb-in.

ALLOWABLE UNIFORM LOAD q

$$M_{max} = \frac{(q + q_0)L^2}{2} \qquad q_{allow} + q_0 = \frac{2M_{allow}}{L^2}$$

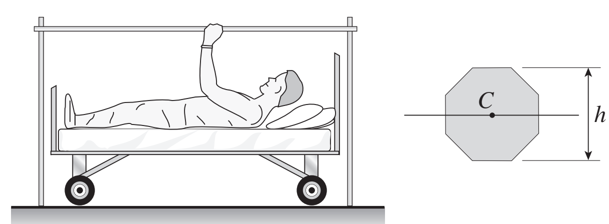
$$q_{allow} = \frac{2M_{allow}}{L^2} - q_0 = \frac{2(29{,}750\text{ lb-in.})}{(36\text{ in.})^2} - 2.5\text{ lb/in.}$$

$$= 45.91 - 2.5 = 43.41\text{ lb/in.}$$

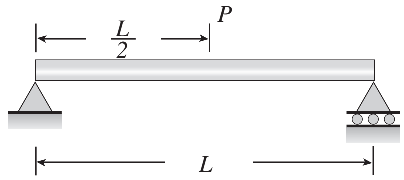
$$q_{allow} = (43.41)(12) = 521\text{ lb/ft} \longleftarrow$$

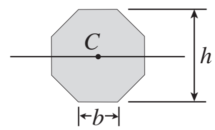
Problem 5.6-10 A so-called "trapeze bar" in a hospital room provides a means for patients to exercise while in bed (see figure). The bar is 2.1 m long and has a cross section in the shape of a regular octagon. The design load is 1.2 kN applied at the midpoint of the bar, and the allowable bending stress is 200 MPa.

Determine the minimum height *h* of the bar. (Assume that the ends of the bar are simply supported and that the weight of the bar is negligible.)



Solution 5.6-10 Trapeze bar (regular octagon)





$P = 1.2$ kN $\quad L = 2.1$ m $\quad \sigma_{allow} = 200$ MPa

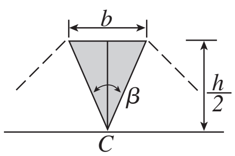
Determine minimum height h.

MAXIMUM BENDING MOMENT

$$M_{max} = \frac{PL}{4} = \frac{(1.2 \text{ kN})(2.1 \text{ m})}{4} = 630 \text{ N} \cdot \text{m}$$

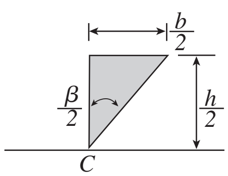
PROPERTIES OF THE CROSS SECTION

Use Appendix D, Case 25, with $n = 8$



b = length of one side

$$\beta = \frac{360°}{n} = \frac{360°}{8} = 45°$$



$$\tan\frac{\beta}{2} = \frac{b}{h} \text{ (from triangle)}$$

$$\cot\frac{\beta}{2} = \frac{h}{b}$$

For $\beta = 45°$: $\dfrac{b}{h} = \tan\dfrac{45°}{2} = 0.41421$

$$\frac{h}{b} = \cot\frac{45°}{2} = 2.41421$$

MOMENT OF INERTIA

$$I_C = \frac{nb^4}{192}\left(\cot\frac{\beta}{2}\right)\left(3\cot^2\frac{\beta}{2} + 1\right)$$

$$I_C = \frac{8b^4}{192}(2.41421)[3(2.41421)^2 + 1] = 1.85948b^4$$

$b = 0.41421h \quad \therefore I_C = 1.85948(0.41421h)^4 = 0.054738h^4$

SECTION MODULUS

$$S = \frac{I_C}{h/2} = \frac{0.054738h^4}{h/2} = 0.109476h^3$$

MINIMUM HEIGHT h

$$\sigma = \frac{M}{S} \qquad S = \frac{M}{\sigma}$$

$$0.109476h^3 = \frac{630\text{N} \cdot \text{m}}{200 \text{ MPa}} = 3.15 \times 10^{-6} \text{ m}^3$$

$h^3 = 28.7735 \times 10^{-6}$ m$^3 \quad h = 0.030643$ m

$\therefore h_{min} = 30.6$ mm ←

ALTERNATIVE SOLUTION ($n = 8$)

$$M = \frac{PL}{4} \quad \beta = 45° \quad \tan\frac{\beta}{2} = \sqrt{2} - 1 \quad \cot\frac{\beta}{2} = \sqrt{2} + 1$$

$$b = (\sqrt{2} - 1)h \quad h = (\sqrt{2} + 1)b$$

$$I_C = \left(\frac{11 + 8\sqrt{2}}{12}\right)b^4 = \left(\frac{4\sqrt{2} - 5}{12}\right)h^4$$

$$S = \left(\frac{4\sqrt{2} - 5}{6}\right)h^3 \quad h^3 = \frac{3PL}{2(4\sqrt{2} - 5)\sigma_{allow}} \quad ←$$

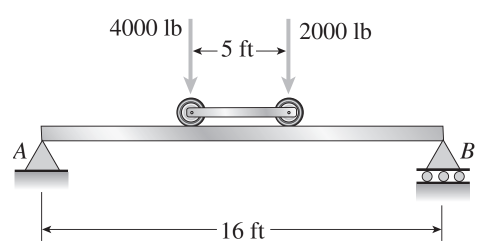
Substitute numerical values:

$h^3 = 28.7735 \times 10^{-6}$ m$^3 \quad h_{min} = 30.643$ mm ←

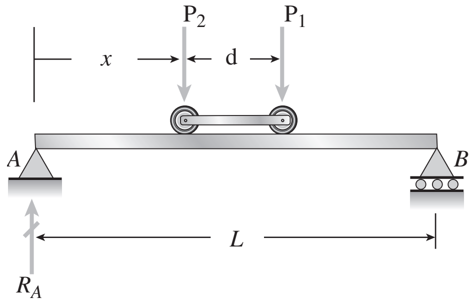
Problem 5.6-11 A two-axle carriage that is part of an overhead traveling crane in a testing laboratory moves slowly across a simple beam AB (see figure). The load transmitted to the beam from the front axle is 2000 lb and from the rear axle is 4000 lb. The weight of the beam itself may be disregarded.

(a) Determine the minimum required section modulus S for the beam if the allowable bending stress is 15.0 ksi, the length of the beam is 16 ft, and the wheelbase of the carriage is 5 ft.

(b) Select a suitable I-beam (S shape) from Table E-2, Appendix E.



Solution 5.6-11 Moving carriage



P_1 = load on front axle
= 2000 lb

P_2 = load on rear axle
= 4000 lb

L = 16 ft d = 5 ft σ_{allow} = 15 ksi

x = distance from support A to the larger load P_2 (feet)

$$R_A = P_2\left(\frac{L-x}{L}\right) + P_1\left(\frac{L-x-d}{L}\right)$$

$$= (4000 \text{ lb})\left(1 - \frac{x}{16}\right) + (2000 \text{ lb})\left(1 - \frac{x}{16} - \frac{5}{16}\right)$$

$$= 125(43 - 3x) \quad (x = \text{ft}; R_A = \text{lb})$$

Bending moment under larger load P_2

$$M = R_A\, x = 125(43x - 3x^2) \quad (x = \text{ft}; M = \text{lb-ft})$$

Maximum bending moment

Set $\frac{dM}{dx}$ equal to zero and solve for $x = x_m$.

$$\frac{dM}{dx} = 125(43 - 6x) = 0 \quad x = x_m = \frac{43}{6} = 7.1667 \text{ ft}$$

$$M_{max} = (M)_{x=x_m} = 125\left[(43)\left(\frac{43}{6}\right) - 3\left(\frac{43}{6}\right)^2\right]$$

$$= 19,260 \text{ lb-ft} = 231,130 \text{ lb-in.}$$

(a) Minimum section modulus

$$S_{min} = \frac{M_{max}}{\sigma_{allow}} = \frac{231,130 \text{ lb-in.}}{15,000 \text{ psi}} = 15.41 \text{ in.}^3 \quad \leftarrow$$

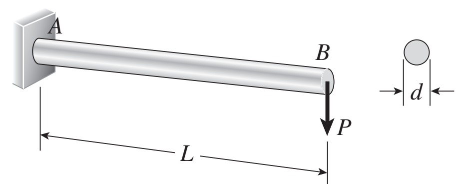
(b) Select on I-beam (S shape)

Table E-2. Select S 8 × 23 ←

(S = 16.2 in.^{3})

Problem 5.6-12 A cantilever beam AB of circular cross section and length L = 450 mm supports a load P = 400 N acting at the free end (see figure). The beam is made of steel with an allowable bending stress of 60 MPa.

Determine the required diameter d_{min} of the beam, considering the effect of the beam's own weight.



Solution 5.6-12 Cantilever beam

Data L = 450 mm P = 400 N

σ_{allow} = 60 MPa

γ = weight density of steel
= 77.0 kN/m^3

Weight of beam per unit length

$$q = \gamma\left(\frac{\pi d^2}{4}\right)$$

Maximum bending moment

$$M_{max} = PL + \frac{qL^2}{2} = PL + \frac{\pi \gamma d^3 L^2}{8}$$

Section modulus $S = \dfrac{\pi d^3}{32}$

Minimum diameter

$$M_{max} = \sigma_{allow} S$$

$$PL + \frac{\pi \gamma d^2 L^2}{8} = \sigma_{allow}\left(\frac{\pi d^3}{32}\right)$$

Rearrange the equation:

$$\sigma_{allow} d^3 - 4\gamma L^2 d^2 - \frac{32\, PL}{\pi} = 0$$

(Cubic equation with diameter d as unknown.)

Substitute numerical values (d = meters):

$$(60 \times 10^6 \text{ N/m}^2)d^3 - 4(77,000 \text{ N/m}^3)(0.45\text{m})^2 d^2$$

$$- \frac{32}{\pi}(400 \text{ N})(0.45 \text{ m}) = 0$$

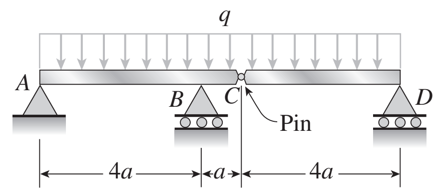
$$60,000d^3 - 62.37d^2 - 1.833465 = 0$$

Solve the equation numerically:

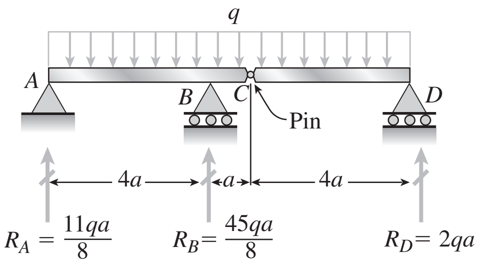
d = 0.031614 m d_{min} = 31.61 mm ←

Problem 5.6-13 A compound beam *ABCD* (see figure) is supported at points *A*, *B*, and *D* and has a splice (represented by the pin connection) at point *C*. The distance $a = 6.0$ ft and the beam is a W 16 × 57 wide-flange shape with an allowable bending stress of 10,800 psi.

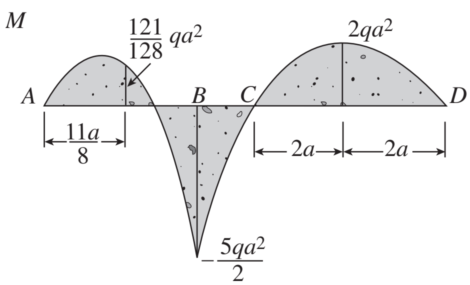
Find the allowable uniform load q_{allow} that may be placed on top of the beam, taking into account the weight of the beam itself.



Solution 5.6-13 **Compound beam**



Pin connection at point C.



$$M_{max} = \frac{5qa^2}{2} = \sigma_{allow} S$$

$$q_{max} = \frac{2\sigma_{allow} S}{5a^2} \qquad q_{allow} = q_{max} - (\text{weight of beam})$$

DATA: $a = 6 \text{ ft} = 72 \text{ in.} \qquad \sigma_{allow} = 10{,}800 \text{ psi}$

W 16 × 57 $\qquad S = 92.2 \text{ in.}^3$

ALLOWABLE UNIFORM LOAD

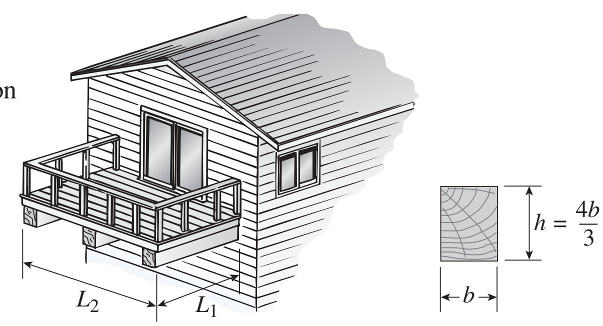
$$q_{max} = \frac{2(10{,}800 \text{ psi})(92.2 \text{ in.}^3)}{5(72 \text{ in.})^2} = 76.833 \text{ lb/in.}$$

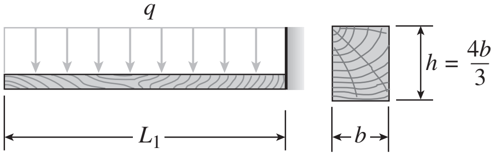
$$= 922 \text{ lb/ft}$$

$$q_{allow} = 922 \text{ lb/ft} - 57 \text{ lb/ft} = 865 \text{ lb/ft} \quad \leftarrow$$

Problem 5.6-14 A small balcony constructed of wood is supported by three identical cantilever beams (see figure). Each beam has length $L_1 = 2.1$ m, width *b*, and height $h = 4b/3$. The dimensions of the balcon floor are $L_1 \times L_2$, with $L_2 = 2.5$ m. The design load is 5.5 kPa acting over the entire floor area. (This load accounts for all loads except the weights of the cantilever beams, which have a weight density $\gamma = 5.5$ kN/m^3.) The allowable bending stress in the cantilevers is 15 MPa.

Assuming that the middle cantilever supports 50% of the load and each outer cantilever supports 25% of the load, determine the required dimensions *b* and *h*.





$L_1 = 2.1$ m $\quad L_2 = 2.5$ m $\quad$ Floor dimensions: $L_1 \times L_2$

Design load $= w = 5.5$ kPa

$\gamma = 5.5$ kN/m^3 (weight density of wood beam)

$\sigma_{allow} = 15$ MPa

MIDDLE BEAM SUPPORTS 50% OF THE LOAD.

$$\therefore\ q = w\left(\frac{L_2}{2}\right) = (5.5 \text{ kPa})\left(\frac{2.5 \text{ m}}{2}\right) = 6875 \text{ N/m}$$

WEIGHT OF BEAM

$$q_0 = \gamma bh = \frac{4\gamma b^2}{3} = \frac{4}{3}(5.5 \text{ kN/m}^2)b^2$$

$$= 7333b^2 \text{ (N/m)} \qquad (b = \text{meters})$$

Maximum bending moment

$$M_{\max} = \frac{(q + q_0)L_1^2}{2} = \frac{1}{2}(6875 \text{ N/m} + 7333b^2)(2.1 \text{ m})^2$$
$$= 15{,}159 + 16{,}170b^2 \ \ (\text{N} \cdot \text{m})$$

$$S = \frac{bh^2}{6} = \frac{8b^3}{27}$$

$$M_{\max} = \sigma_{\text{allow}}\, S$$

$$15{,}159 + 16{,}170b^2 = (15 \times 10^6 \text{ N/m}^2)\left(\frac{8b^3}{27}\right)$$

Rearrange the equation:

$$(120 \times 10^6)b^3 - 436{,}590b^2 - 409{,}300 = 0$$

Solve numerically for dimension b

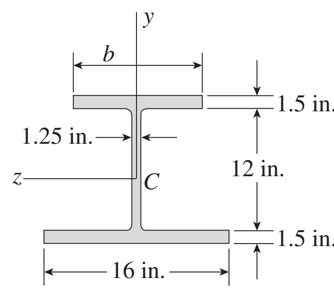
$$b = 0.1517 \text{ m} \qquad h = \frac{4b}{3} = 0.2023 \text{ m}$$

Required dimensions

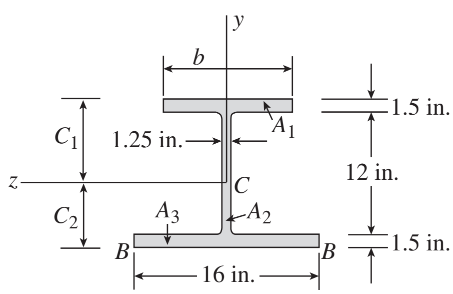
$b = 152$ mm $h = 202$ mm ←

Problem 5.6-15 A beam having a cross section in the form of an unsymmetric wide-flange shape (see figure) is subjected to a negative bending moment acting about the z axis.

Determine the width b of the top flange in order that the stresses at the top and bottom of the beam will be in the ratio 4:3, respectively.



Solution 5.6-15 **Unsymmetric wide-flange beam**



Stresses at top and bottom are in the ratio 4:3.
Find b (inches)

h = height of beam = 15 in.

Locate centroid

$$\frac{\sigma_{top}}{\sigma_{\text{bottom}}} = \frac{c_1}{c_2} = \frac{4}{3}$$

$$c_1 = \frac{4}{7}h = \frac{60}{7} = 8.57143 \text{ in.}$$

$$c_2 = \frac{3}{7}h = \frac{45}{7} = 6.42857 \text{ in.}$$

Areas of the cross section (in.^{2})

$A_1 = 1.5b \qquad A_2 = (12)(1.25) = 15$ in.^{2}

$A_3 = (16)(1.5) = 24$ in.^{2}

$A = A_1 + A_2 + A_3 = 39 + 1.5b$ (in.^{2})

First moment of the cross-sectional area about the lower edge B-B

$$Q_{BB} = \sum \bar{y}_i A_i = (14.25)(1.5b) + (7.5)(15) + (0.75)(24)$$
$$= 130.5 + 21.375b \ \ (\text{in.}^3)$$

Distance c_2 from line B-B to the centroid C

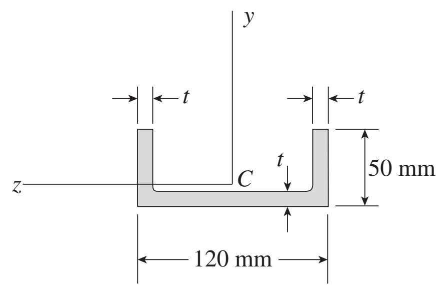
$$c_2 = \frac{Q_{BB}}{A} = \frac{130.5 + 21.375b}{39 + 1.5b} = \frac{45}{7} \text{ in.}$$

Solve for b

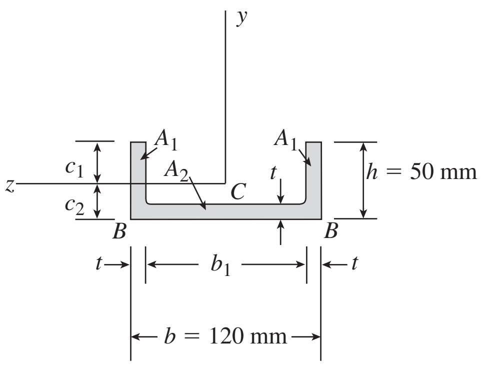
$(39 + 1.5b)(45) = (130.5 + 21.375b)(7)$
$82.125b = 841.5 \qquad b = 10.25$ in. ←

Problem 5.6-16 A beam having a cross section in the form of a channel (see figure) is subjected to a bending moment acting about the z axis.

Calculate the thickness t of the channel in order that the bending stresses at the top and bottom of the beam will be in the ratio 7:3, respectively.



Solution 5.6-16 Channel beam



t = thickness (constant) (t is in millimeters)

$b_1 = b - 2t = 120\text{ mm} - 2t$

Stresses at the top and bottom are in the ratio 7:3.

Determine the thickness t.

LOCATE CENTROID

$$\frac{\sigma_{\text{top}}}{\sigma_{\text{bottom}}} = \frac{c_1}{c_2} = \frac{7}{3}$$

$$c_1 = \frac{7}{10}h = 35\text{ mm}$$

$$c_2 = \frac{3}{10}h = 15\text{ mm}$$

AREAS OF THE CROSS SECTION (mm^2)

$$A_1 = ht = 50t \qquad A_2 = b_1 t = 120t - 2t^2$$

$$A = 2A_1 + A_2 = 220t - 2t^2 = 2t(110-t)$$

FIRST MOMENT OF THE CROSS-SECTIONAL AREA ABOUT THE LOWER EDGE B-B

$$Q_{BB} = \sum y_i A_i = (2)\left(\frac{t}{2}\right)(50t) + \left(\frac{t}{2}\right)(b_1)(t)$$

$$= 2(25)(50t) + \left(\frac{t}{2}\right)(120 - 2t)(t)$$

$$= t(2500 + 60t - t^2) \qquad (t = \text{mm};\ Q = \text{mm}^3)$$

DISTANCE c_2 FROM LINE B-B TO THE CENTROID C

$$c_2 = \frac{Q_{BB}}{A} = \frac{t(2500 + 60t - t^2)}{2t(110 - t)}$$

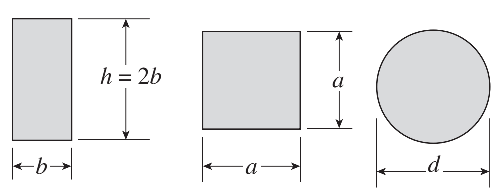
$$= \frac{2500 + 60t - t^2}{2(110 - t)} = 15\text{ mm}$$

SOLVE FOR t

$$2(110 - t)(15) = 2500 + 60t - t^2$$

$$t^2 - 90t + 800 = 0 \qquad t = 10\text{ mm} \quad \longleftarrow$$

Problem 5.6-17 Determine the ratios of the weights of three beams that have the same length, are made of the same material, are subjected to the same maximum bending moment, and have the same maximum bending stress if their cross sections are (1) a rectangle with height equal to twice the width, (2) a square, and (3) a circle (see figures).



Solution 5.6-17 Ratio of weights of three beams

Beam 1: Rectangle ($h = 2b$)
Beam 2: Square (a = side dimension)
Beam 3: Circle (d = diameter)
L, γ, M_{max}, and σ_{max} are the same in all three beams.

S = section modulus $\quad S = \dfrac{M}{\sigma}$

Since M and σ are the same, the section moduli must be the same.

(1) RECTANGLE: $S = \dfrac{bh^2}{6} = \dfrac{2b^3}{3} \qquad b = \left(\dfrac{3S}{2}\right)^{1/3}$

$$A_1 = 2b^2 = 2\left(\frac{3S}{2}\right)^{2/3} = 2.6207\,S^{2/3}$$

(2) SQUARE: $S = \dfrac{a^3}{6} \qquad a = (6S)^{1/3}$

$$A_2 = a^2 = (6S)^{2/3} = 3.3019\,S^{2/3}$$

(3) CIRCLE: $S = \dfrac{\pi d^3}{32} \qquad d = \left(\dfrac{32\,S}{\pi}\right)^{1/3}$

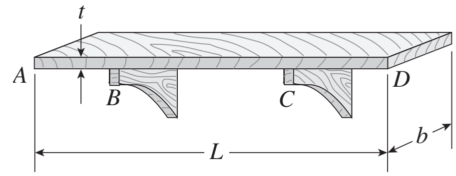
$$A_3 = \frac{\pi d^2}{4} = \frac{\pi}{4}\left(\frac{32S}{\pi}\right)^{2/3} = 3.6905\,S^{2/3}$$

Weights are proportional to the cross-sectional areas (since L and γ are the same in all 3 cases).

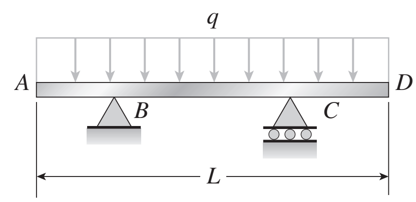
$W_1 : W_2 : W_3 = A_1 : A_2 : A_3$
$A_1 : A_2 : A_3 = 2.6207 : 3.3019 : 3.6905$
$W_1 : W_2 : W_3 = 1 : 1.260 : 1.408$ ←

Problem 5.6-18 A horizontal shelf AD of length L = 900 mm, width b = 300 mm, and thickness t = 20 mm is supported by brackets at B and C [see part (a) of the figure]. The brackets are adjustable and may be placed in any desired positions between the ends of the shelf. A uniform load of intensity q, which includes the weight of the shelf itself, acts on the shelf [see part (b) of the figure].

Determine the maximum permissible value of the load q if the allowable bending stress in the shelf is σ_{allow} = 5.0 MPa and the position of the supports is adjusted for maximum load-carrying capacity.

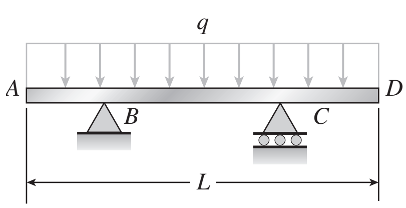


(a)

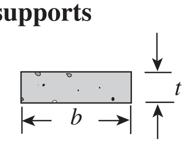


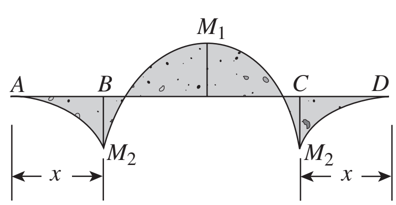
(b)

Solution 5.6-18 Shelf with adjustable supports



(a)





(b)

L = 900 mm
b = 300 mm
t = 20 mm
σ_{allow} = 5.0 MPa

For maximum load-carrying capacity, place the supports so that $M_1 = |M_2|$.

Let x = length of overhang

$$M_1 = \frac{qL}{8}(L - 4x) \qquad |M_2| = \frac{qx^2}{2}$$

$$\therefore\ \frac{qL}{8}(L - 4x) = \frac{qx^2}{2}$$

Solve for x: $x = \dfrac{L}{2}(\sqrt{2} - 1)$

Substitute x into the equation for either M_1 or $|M_2|$:

$$M_{max} = \frac{qL^2}{8}(3 - 2\sqrt{2}) \qquad \text{Eq. (1)}$$

$$M_{max} = \sigma_{allow}\,S = \sigma_{allow}\left(\frac{bt^2}{6}\right) \qquad \text{Eq. (2)}$$

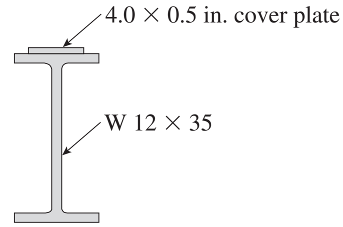
Equate M_{max} from Eqs. (1) and (2) and solve for q:

$$q_{max} = \frac{4bt^2\sigma_{allow}}{3L^2(3 - 2\sqrt{2})}$$

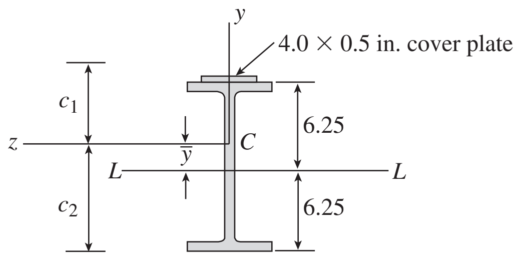
Substitute numerical values:
q_{max} = 5.76 kN/m ←

Problem 5.6-19 A steel plate (called a *cover plate*) having cross-sectional dimensions 4.0 in. × 0.5 in. is welded along the full length of the top flange of a W 12 × 35 wide-flange beam (see figure, which shows the beam cross section).

What is the percent increase in section modulus (as compared to the wide-flange beam alone)?



Solution 5.6-19 **Beam with cover plate**



All dimensions in inches.

WIDE-FLANGE BEAM ALONE
(AXIS L-L IS CENTROIDAL AXIS)

W 12 × 35 $d = 12.50$ in.
$A_0 = 10.3 \text{ in.}^2 \quad I_0 = 2.85 \text{ in.}^4 \quad S_0 = 45.6 \text{ in.}^3$

BEAM WITH COVER PLATE
(z AXIS IS CENTROIDAL AXIS)

$A_1 = A_0 + (4.0 \text{ in.})(0.5 \text{ in.}) = 12.3 \text{ in.}^2$

First moment with respect to axis L-L:

$$Q_1 = \sum \bar{y}_i A_i = (6.25 \text{ in.} + 0.25 \text{ in.})(4.0 \text{ in.})(0.5 \text{ in.}) = 13.00 \text{ in.}^3$$

$$\bar{y} = \frac{Q_1}{A_1} = \frac{13.00 \text{ in.}^3}{12.3 \text{ in.}^2} = 1.057 \text{ in.}$$

$$c_1 = 6.25 + 0.5 - \bar{y} = 5.693 \text{ in.}$$

$$c_2 = 6.25 + \bar{y} = 7.307 \text{ in.}$$

Moment of inertia about axis L-L:

$$I_{L-L} = I_0 + \frac{1}{12}(4.0)(0.5)^3 + (4.0)(0.5)(6.25 + 0.25)^2 = 369.5 \text{ in.}^4$$

Moment of inertia about z axis:

$$I_{L-L} = I_z + A_1\bar{y}^2 \quad I_z = I_{1-1} - A_1\bar{y}^2$$

$$I_z = 369.5 \text{ in.}^4 - (12.3 \text{ in.}^2)(1.057 \text{ in.})^2 = 355.8 \text{ in.}^4$$

SECTION MODULUS (Use the smaller of the two section moduli)

$$S_1 = \frac{I_z}{c_2} = \frac{355.8 \text{ in.}^4}{7.307 \text{ in.}} = 48.69 \text{ in.}^3$$

INCREASE IN SECTION MODULUS

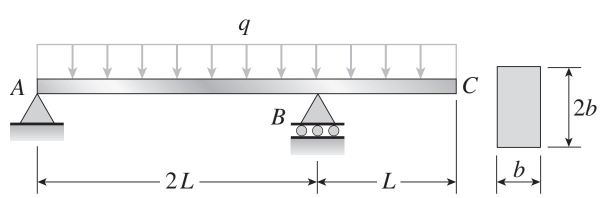
$$\frac{S_1}{S_0} = \frac{48.69}{45.6} = 1.068$$

Percent increase = 6.8% ←

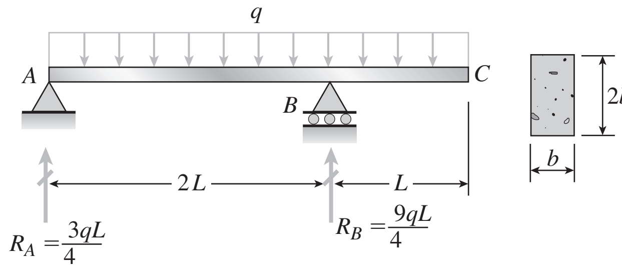
Problem 5.6-20 A steel beam ABC is simply supported at A and B and has an overhang BC of length $L = 150$ mm (see figure on the next page). The beam supports a uniform load of intensity $q = 3.5$ kN/m over its entire length of 450 mm. The cross section of the beam is rectangular with width b and height $2b$. The allowable bending stress in the steel is $\sigma_{allow} = 60$ MPa and its weight density is $\gamma = 77.0 \text{ kN/m}^3$.

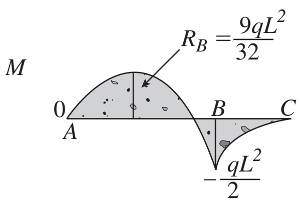
(a) Disregarding the weight of the beam, calculate t he required width b of the rectangular cross section.

(b) Taking into account the weight of the beam, calculate the required width b.



Solution 5.6-20 **Beam with an overhang**





$L = 150$ mm
$q = 3.5$ kN/m
$\sigma_{allow} = 60$ MPa
$\gamma = 77.0$ kN/m^3

$$M_{max} = \frac{qL^2}{2} \qquad S = \frac{bh^2}{6} = \frac{2b^3}{3}$$

(a) Disregard the weight of the beam

$$M_{max} = \sigma_{allow} S \qquad \frac{qL^2}{2} = \sigma_{allow}\left(\frac{2b^3}{3}\right)$$

$$b^3 = \frac{3qL^2}{4\sigma_{allow}}$$

Substitute numerical values:

$$b^3 = \frac{3(3.5 \text{ kN/m})(150 \text{ mm})^2}{4(60 \text{ MPa})} = 0.98438 \times 10^{-6} \text{ m}^3$$

$b = 0.00995$ m $= 9.95$ mm ←

(b) Include the weight of the beam

$q_0 =$ weight of beam per unit length
$q_0 = \gamma(b)(2b) = 2\gamma b^2$

$$M_{max} = \frac{(q - q_0)L^2}{2} = \frac{1}{2}(q + 2\gamma b^2)L^2$$

$$S = \frac{2b^3}{3} \qquad M_{max} = \sigma_{allow} S$$

$$\frac{1}{2}(q + 2\gamma b^2)L^2 = \sigma_{allow}\left(\frac{2b^3}{3}\right)$$

Rearrange the equation:

$$4\sigma_{allow} b^3 - 6\gamma L^2 b^2 - 3qL^2 = 0$$

Substitute numerical values:

$$(240 \times 10^6)b^3 - 10{,}395b^2 - 236.25 = 0$$

($b =$ meters)

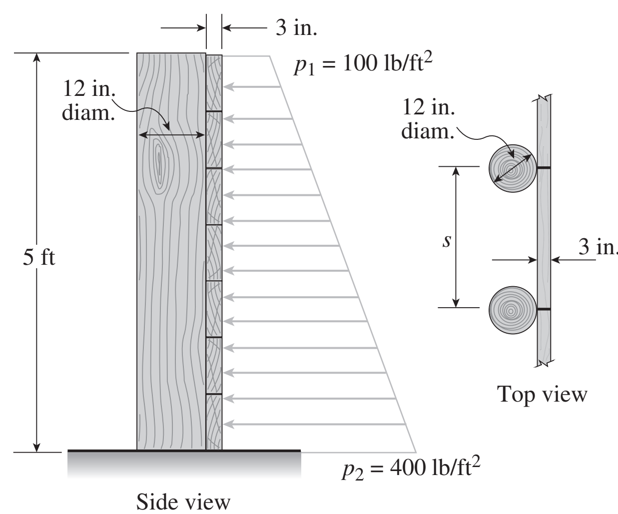
Solve the equation:

$b = 0.00996$ m $= 9.96$ mm ←

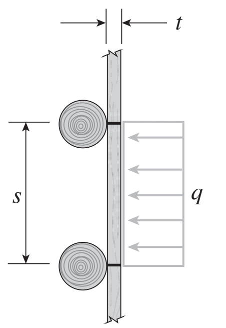
Problem 5.6-21 A retaining wall 5 ft high is constructed of horizontal wood planks 3 in. thick (actual dimension) that are supported by vertical wood piles of 12 in. diameter (actual dimension), as shown in the figure. The lateral earth pressure is $p_1 = 100$ lb/ft^2 at the top of the wall and $p_2 = 400$ lb/ft^2 at the bottom.

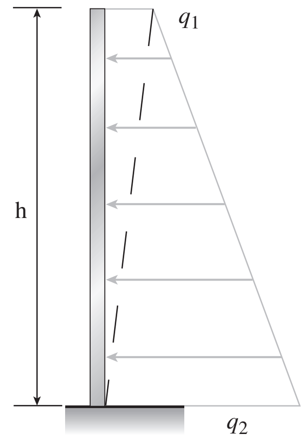
Assuming that the allowable stress in the wood is 1200 psi, calculate the maximum permissible spacing s of the piles.

(*Hint:* Observe that the spacing of the piles may be governed by the load-carrying capacity of either the planks or the piles. Consider the piles to act as cantilever beams subjected to a trapezoidal distribution of load, and consider the planks to act as simple beams between the piles. To be on the safe side, assume that the pressure on the bottom plank is uniform and equal to the maximum pressure.)



Solution 5.6-21 **Retaining wall**





(1) PLANK AT THE BOTTOM OF THE DAM

t = thickness of plank = 3 in.

b = width of plank (perpendicular to the plane of the figure)

p_2 = maximum soil pressure = 400 lb/ft^2 = 2.778 lb/in.^{2}

s = spacing of piles

$q = p_2 b$ $\quad \sigma_{allow} = 1200$ psi $\quad S$ = section modulus

$$M_{max} = \frac{qs^2}{8} = \frac{p_2 bs^2}{8} \qquad S = \frac{bt^2}{6}$$

$$M_{max} = \sigma_{allow} S \quad \text{or} \quad \frac{p_2 bs^2}{8} = \sigma_{allow}\left(\frac{bt^2}{6}\right)$$

Solve for s:

$$s = \sqrt{\frac{4\sigma_{allow} t^2}{3p_2}} = 72.0 \text{ in.}$$

(2) VERTICAL PILE

h = 5 ft = 60 in.

p_1 = soil pressure at the top = 100 lb/ft^2 = 0.6944 lb/in.^{2}

$q_1 = p_1 s$

$q_2 = p_2 s$

d = diameter of pile = 12 in.

Divide the trapezoidal load into two triangles (see dashed line).

$$M_{max} = \frac{1}{2}(q_1)(h)\left(\frac{2h}{3}\right) + \frac{1}{2}(q_2)(h)\left(\frac{h}{3}\right) = \frac{sh^2}{6}(2p_1 + p_2)$$

$$S = \frac{\pi d^3}{32} \qquad M_{max} = \sigma_{allow} S \quad \text{or}$$

$$\frac{sh^2}{6}(2p_1 + p_2) = \sigma_{allow}\left(\frac{\pi d^3}{32}\right)$$

Solve for s:

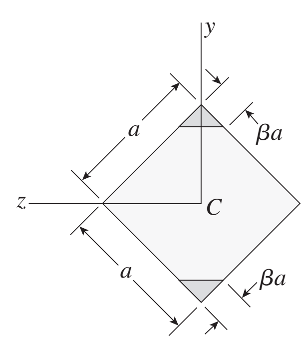
$$s = \frac{3\pi\sigma_{allow} d^3}{16h^2(2p_1 + p_2)} = 81.4 \text{ in.}$$

PLANK GOVERNS $\quad s_{max} = 72.0$ in. ←

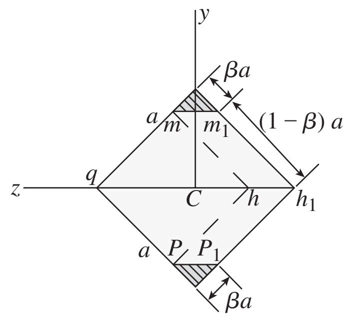
Problem 5.6-22 A beam of square cross section (a = length of each side) is bent in the plane of a diagonal (see figure). By removing a small amount of material at the top and bottom corners, as shown by the shaded triangles in the figure, we can increase the section modulus and obtain a stronger beam, even though the area of the cross section is reduced.

(a) Determine the ratio β defining the areas that should be removed in order to obtain the strongest cross section in bending.

(b) By what percent is the section modulus increased when the areas are removed?



Solution 5.6-22 **Beam of square cross section with corners removed**



a = length of each side
β_a = amount removed
Beam is bent about the z axis.

Entire cross section (Area D)

$$I_0 = \frac{a^4}{12} \quad c_0 = \frac{a}{\sqrt{2}} \quad S_0 = \frac{I_0}{c_0} = \frac{a^3\sqrt{2}}{12}$$

Square mnpq (Area 1)

$$I_1 = \frac{(1-\beta)^4 a^4}{12}$$

Parallelogram mm, n, n (Area 2)

$$I_2 = \frac{1}{3}\,(\text{base})(\text{height})^3$$

$$I_2 = \frac{1}{3}(\beta a\sqrt{2})\left[\frac{(1-\beta)a}{\sqrt{2}}\right]^3 = \frac{\beta a^4}{6}(1-\beta)^3$$

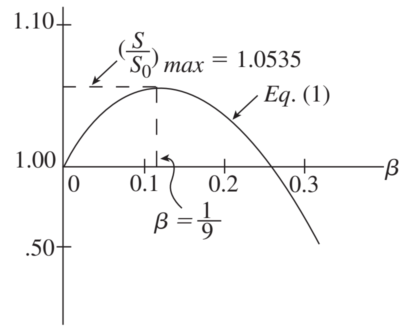
Reduced cross section (Area qmm, m, p, pq)

$$I = I_1 + 2I_2 = \frac{a^4}{12}(1+3\beta)(1-\beta)^3$$

$$c = \frac{(1-\beta)a}{\sqrt{2}} \qquad S = \frac{I}{c} = \frac{\sqrt{2}\,a^3}{12}(1+3\beta)(1-\beta)^2$$

Ratio of section moduli

$$\frac{S}{S_0} = (1+3\beta)(1-\beta)^2 \qquad \text{Eq. (1)}$$



Graph of Eq. (1)

(a) Value of β for a maximum value of S/S_0

$$\frac{d}{d\beta}\left(\frac{S}{S_0}\right) = 0$$

Take the derivative and solve this equation for β.

$$\beta = \frac{1}{9} \quad \longleftarrow$$

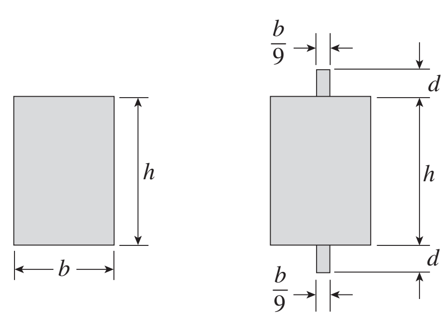
(b) Maximum value of S/S_0

Substitute $\beta = 1/9$ into Eq. (1). $(S/S_0)_{max} = 1.0535$

The section modulus is increased by 5.35% when the triangular areas are removed. ⟵

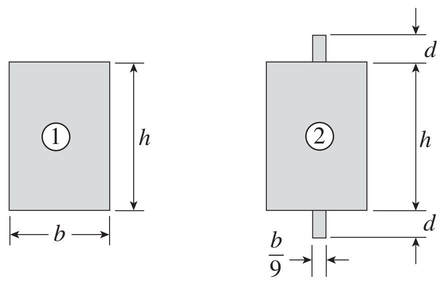
Problem 5.6-23 The cross section of a rectangular beam having width b and height h is shown in part (a) of the figure. For reasons unknown to the beam designer, it is planned to add structural projections of width $b/9$ and height d to the top and bottom of the beam [see part (b) of the figure].

For what values of d is the bending-moment capacity of the beam increased? For what values is it decreased?



(a) (b)

Solution 5.6-23 Beam with projections



(a) (b)

(1) ORIGINAL BEAM

$$I_1 = \frac{bh^3}{12} \qquad c_1 = \frac{h}{2} \qquad S_1 = \frac{I_1}{c_1} = \frac{bh^2}{6}$$

(2) BEAM WITH PROJECTIONS

$$I_2 = \frac{1}{12}\left(\frac{8b}{9}\right)h^3 + \frac{1}{12}\left(\frac{b}{9}\right)(h + 2d)^3$$

$$= \frac{b}{108}[8h^3 + (h + 2d)^3]$$

$$c_2 = \frac{h}{2} + d = \frac{1}{2}(h + 2d)$$

$$S_2 = \frac{I_2}{c_2} = \frac{b[8h^3 + (h + 2d)^3]}{54(h + 2d)}$$

RATIO OF SECTION MODULI

$$\frac{S_2}{S_1} = \frac{b[8h^3 + (h + 2d)^3]}{9(h + 2d)(bh^2)} = \frac{8 + \left(1 + \frac{2d}{h}\right)^3}{9\left(1 + \frac{2d}{h}\right)}$$

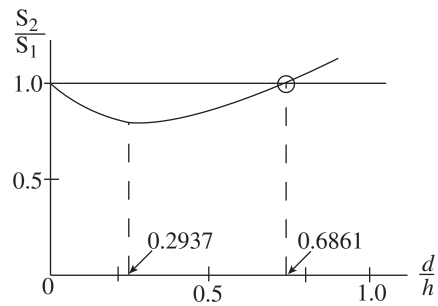
EQUAL SECTION MODULI

Set $\frac{S_2}{S_1} = 1$ and solve numerically for $\frac{d}{h}$.

$$\frac{d}{h} = 0.6861 \quad \text{and} \quad \frac{d}{h} = 0$$

Graph of $\frac{S_2}{S_1}$ versus $\frac{d}{h}$

$\frac{d}{h}$	$\frac{S_2}{S_1}$
0	1.000
0.25	0.8426
0.50	0.8889
0.75	1.0500
1.00	1.2963



Moment capacity is increased when

$\frac{d}{h} > 0.6861$ ←

Moment capacity is decreased when

$\frac{d}{h} < 0.6861$ ←

NOTES:

$$\frac{S_2}{S_1} = 1 \text{ when } \left(1 + \frac{2d}{h}\right)^3 - 9\left(1 + \frac{2d}{h}\right) + 8 = 0$$

or $\quad \frac{d}{h} = 0.6861$ and 0

$$\frac{S_2}{S_1} \text{ is minimum when } \frac{d}{h} = \frac{\sqrt[3]{4} - 1}{2} = 0.2937$$

↓

$$\left(\frac{S_2}{S_1}\right)_{\min} = 0.8399$$