Financing and access in cooperatives ☆

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Abstract

Cooperative undertakings account for a substantial share of developed market economies and that share is likely to grow with the advent of the new economy. The paper develops a dynamic investment framework that relates access policies, financing and growth of cooperatives. It shows how discriminating among users affects the viability of cooperatives and impacts social efficiency. It then argues that in most circumstances, the cooperative form, even when viable on a stand-alone basis, is a weak competitor against alternative organizational forms. Last, the paper stresses that access policies involve a standard social trade-off between static efficiency and innovation.

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1. Introduction

Cooperative undertakings account for a substantial share of developed market economies. As documented by Hansmann (1996), even in the United States cooperatives dominate or at least figure prominently in a number of industries, such as agriculture, credit cards, hardware, moving companies, electricity and the financial sector. Related forms of cooperative undertakings include joint ventures (R&D joint ventures, Intelsat, airline seat reservation systems), consortia (undersea fiber optic cable systems), and partnerships (law firms, investment banks). Cooperatives may

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1 For example, cooperatives market 32% of the products produced and processed in the agri-food chain.

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become even more prominent with the advent of the new economy. For example, an important question confronting firms and antitrust authorities is whether standards and B2B exchanges should be controlled by a single user, or by a community of users, whose extent then has to be defined. Another case in point is patent pools, which have played a crucial role in the aerospace and automobile industries as well as in a number of other industries.

The existence of cooperative undertakings can usually be traced to two forms of returns to scale in the provision of an input and to the concomitant eagerness of the users to protect themselves from monopoly behavior. First, returns to scale may be associated with large fixed costs. As Hansmann (1996, 1999) argues, the capital intensity of equipment manufacturing is often and incorrectly thought of as an important barrier to the emergence of cooperatives. For example, some of the largest farm supply cooperatives in the US run very capital-intensive operations (oil refining, seeds, fertilizers,...); credit card cooperatives involve substantial sunk investment in telecommunications networks, software and branding. Because they are shared among the users, fixed costs in cooperatives give rise to “cost-sharing network externalities”. The second form of returns to scale relates to “classical network externalities”. Such network externalities arise for example for credit cards, moving companies, flower delivery services or alliances around a standard.

This paper focuses on the financing of cooperatives: How do cooperatives manage their financing and growth? Do they have the proper investment incentives? When are they viable? Cooperatives rely primarily on the proportional assessments levied on their members' usage of the facilities (the “unit retains” that are kept after patronage dividends are redistributed to the members) and equity investments by the members. They by and large have little or no access to external finance.

The viability of cooperatives and their investment incentives are closely related to the cooperatives' access policies. Some cooperatives essentially do not discriminate between incumbents and new or expanding members. Most however practice such discrimination in various ways. Usage fees may decrease with past cumulative usage, or may depend on the user’s status (e.g., internal vs external in a patent pool). Cooperatives may ask for entry fees or may allow older

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2 See Merges (1996) for an overview of these pools, as well as those related to collective rights organizations (such as the American Society of Composers, Authors and Publishers), Shapiro (2001) for a discussion of pools in the context of semiconductors and standard setting, and Lerner and Tirole (2004) for theoretical modelling of pool formation and characteristics.

3 Well-known patent pools in the new economy include the MPEG LA pool (protocol for compressing and transmitting digitalized audio and video signals) and the former Cylink-RSA public key encryption pool that for a while defined a de facto proprietary standard. Still, patent pools have not yet had the impact on the new economy that might be expected from the observation that software, semiconductor and biotechnology products potentially infringe on thousands of intellectual property rights, so that pooling would ceteris paribus appear very desirable. Moreover, when pooling occurs it often takes the more limited form of cross licensing. Industry participants’ concerns about how the pool will evolve and antitrust authorities’ fear of improper use of pools to collude or erect barriers to entry may account for this currently low diffusion of pools.

4 There are literal exceptions to this characterization. In the US tax-exempt nonprofit hospitals have been able (until the 1986 Tax Reform Act) to borrow to help cover substantial capital investments. But such cooperatives had access to collateral (relatively safe income streams, building, brandname) and, as we said, benefited from borrowing subsidies. The fact that many of these hospitals had trouble raising funds to cover their investment needs and converted to for-profit status in the 90’s confirms the low access of cooperatives to external financing.

5 See, e.g. Hansmann (1996) and Rathbone and Wissman (1993). For example, under a per-unit capital returns system, new members or members who increase their consumption must make new investments to reach a target capital-patronage ratio. Redemption programs include first-in, first-out redemptions, base capital methods (redeeming overinvested patrons), and percent-of-all-equities programs (redemptions proportional to outstanding equities).

6 For example, the Microelectronics and Computer Technology Corporation (MCC), a large-scale IT project involving 21 participants had an entry fee of $150,000 at the onset (1983). Newcomers paid a $1,000,000 entry fee in 1986.
members to redeem their shares when they depart.⁷ And many of the “new generation” US farm cooperatives issue transferable and appreciable equity shares, which enable incumbents to recoup some of their investment when departing the venture (Cook and Iliopoulos, 1999).

Access policies matter for both business and antitrust reasons. Liberal access policies allow new members to free ride on previous investments; as we will see, such policies may prevent the venture from getting off the ground and they further encourage short-termism in investment decisions (an issue known as the “horizon problem” in the policy literature). At the other end of the spectrum, very restrictive access policies raise two concerns. First, they may excessively tax newcomers and make the venture underinclusive. Second, when the members compete on the product market, access policies may be used as a barrier to entry. Access policies therefore must strike the right balance between the protection of investment and openness.

To the best of our knowledge, there has been no analytical treatment of the issues covered in this paper. The theoretical literature on cooperatives⁸ focuses on corporate governance and conflict issues and is not cast in a dynamic investment framework. We will however point at some links between our work and two apparently distinct fields: the political economy of social security reform and public utility regulation.

The paper develops a simple overlapping-generations (OLG) framework in order to capture the intergenerational conflicts between incumbents and new members. Investments are financed from assessments or from equity contributions levied from current members. Our study proceeds in a gradual manner in order to identify in a clean way the relevant trade-offs. It focuses first on the intergenerational conflicts by ignoring downstream competition. That is, the members of the cooperative interact only through their membership. In this framework we ask three groups of questions: When are cooperatives viable and how is their investment affected by the absence of discrimination (Section 2)? Are cooperatives robust to competition from other cooperatives, discriminatory or non-discriminatory, and from for-profit companies, and how do cooperatives emerge in an environment in which alternative institutional forms are available (Section 3)? Are cooperatives over or underinclusive and should the level of discrimination between old and new users vary over time (Section 4)? The analysis is then generalized to allow membership value to be eroded by the entry of new, competing members and the paper studies the anticompetitive concerns associated with alternative access policies (Section 5). Last, Section 6 concludes.

2. Are cooperatives viable?

2.1. Model

To model the arrival of new agents/potential users and study access policies, we consider an OLG framework.⁹ Time is discrete, and the horizon infinite, \( t = 0, 1, 2, \ldots \). Agents live for two

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⁷ Although, as noted by Hansmann (1999), redemptions are not as widespread as one might have expected. Rathbone and Wissman (1993), in their study of the various forms of redemption in agricultural cooperatives, document “special equity redemption programs,” that redeem equity to existing members (in agriculture, exit from a cooperative is probably less subject to opportunistic behavior than in other industries). Common programs include payments to a member’s estate after her death, age-of-patron/retirement programs, and disability programs. As one would expect, there are fewer retirement-from-farming or move-away programs.


⁹ The OLG model is a standard, Samuelsonian one. It is easy to generalize our results using, say, a Blanchard (1985) framework, in which agents’ retirement follows a Poisson process. Similarly, the extension to time-dependent populations \( (n_0, n_1, \ldots, n_t) \) is completely straightforward. It predicts a slower transition to the steady state than depicted here if the sequence \( n_t \) is increasing. It can also be used to study the mirror image of a declining membership.
consecutive periods. Each period, a generation with a large number, namely a continuum of mass 1, of agents is born, and an equal-size generation exits. The discount factor is denoted by $\delta<1$.

At each date $t-1$, an investment, costing $I>0$ if incurred, is available, that brings about a technology for date $t$. This technology generates gross surplus $\theta$ for any agent who has access at $t$ to the technology. This surplus can be thought of as the user’s surplus increment relative to publicly available technologies at date $t$. The technology is a public good in that its ex post marginal cost is zero, and furthermore the individual gross surplus $\theta$ is independent of the number of agents who have access to the technology at $t$. Our formulation also implies that agents do not compete on the product market; otherwise, the individual surplus would depend on the access policy. As discussed in the Introduction, we want to separate intergenerational conflicts from competitive issues and therefore delay the introduction of product market competition. Agents are risk neutral and do not face credit constraints. We will further assume that the investment is socially beneficial.

This section assumes that all agents have the same gross surplus $\theta$ from having access to the technology. [Section 4 will consider the case of within-generation heterogeneity and will look at inclusiveness.] The condition that the investment is socially beneficial is then:

$$2\delta\theta > I.$$  \hspace{1cm} (1)

We will analyze the following institutions:

**Investor-owned corporation** (IOC). The investor-owned corporation is externally financed. Provided it has invested at date $t-1$, the IOC rents/licenses its technology at access charge $a_t$ at date $t$.

**Nondiscriminatory cooperative** (NDC). The nondiscriminatory cooperative is the purest form of cooperative: There is no entry fee, no redemption rights and all users of the cooperative at a given date $t$ pay the same amount $a_t$ for the right to use the input produced by the cooperative. We assume that control rights over the investment decision in a cooperative (discriminatory or not) are allocated to the young members — otherwise investment would never take place.

**Fully discriminatory cooperative** (FDC). In a discriminatory cooperative, new members do not pay the same amount as established members. A fully discriminatory cooperative completely disconnects the assessments paid by the established members and by the newcomers. Without loss of generality (see below), we will formalize discrimination as the existence of an entry fee $E_t$ to be paid by new members at date $t$ and chosen by established members. The newcomers’ entry fee is used to defray the investment cost $I$. By convention, new members otherwise pay the same usage price $a_t$ as established members. [In this interpretation, the old-timers do not receive any equity redemption payment when they leave. They benefit from the newcomers’ entry fee through the reduction in the access charge.]

We will investigate the viability and the efficiency of these organizational forms: we will say that an organizational form is **viable** when the socially desirable investment is made in each period, and that it is **efficient** if the technology is moreover accessible to all the users who can benefit from it.

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10 Throughout the analysis, we focus on investment costs and ignore variable operating costs. In practice, the access charge $a_t$ should be interpreted as the amount that needs to be added to variable costs in order to recover the fixed costs of investment.
2.2. Viability in the absence of a competing platform

We will assume here that there is no threat of entry by a rival platform (Section 3 studies upstream competition); as we will see, even in the absence of such competition NDCs are more fragile than the other two institutions.

2.2.1. Investor-owned corporation

Provided it has invested at \( t-1 \), the IOC sets access charge at date \( t \) so as to capture user surplus:

\[
a_t = a^m = \theta,
\]

and so its intertemporal profit is

\[
V^m = \frac{2\delta \theta - I}{1 - \delta}.
\]

Under condition (1), investment is viable in an IOC, since the latter captures the entire social surplus under user homogeneity.

2.2.2. Fully discriminatory cooperative

A fully discriminatory cooperative can also extract the new members’ entire surplus and thus replicates the outcome of an IOC. To see this, first consider the steady state of an FDC and let \( \nu^{FDC} = \theta - a^{FDC} \) denote the old members’ equilibrium instantaneous payoff. In any period, the access charge is linked to the entry fee \( E \) by \( 2a = I - E \).

The old generation optimally seeks to maximize its payoff,

\[
\theta - a = \theta - \frac{I - E}{2};
\]

it thus chooses the largest fee that the young generation is willing to pay:

\[
E = \theta - a + \delta \nu^{FDC},
\]

and gets

\[
V = \theta - a = 2\theta - I + \delta \nu^{FDC}.
\]

That is, the old generation extracts the entire instantaneous surplus generated by the cooperative, plus the discounted rent that the young generation will get later on. In equilibrium\(^1\)

\[
V = \nu^{FDC} = \frac{2\delta \theta - I}{1 - \delta}.
\]

The first generation (born at date 0) extracts the entire surplus and gets

\[
U_0^{FDC} = -I + \delta \nu^{FDC} = \frac{2\delta \theta - I}{1 - \delta} = V^m.
\]

\(^1\) In the steady-state, the entry fee is equal to \( E^{FDC} = \frac{2\delta}{1 - \delta} (2\theta - I) \) while the access charge, \( a^{FDC} = [I - (1 + \delta)\theta]/(1 - \delta) \), is negative. This access charge should however be interpreted as a reduction in the usage fee charged for the input supplied by the cooperative, which is normalized to zero in our framework. In addition, if the demand for the input is variable, non-linear (e.g., two-part) tariffs should be used and the subsidy should be applied to the fixed part of the tariff in order to avoid distortions.
while the later generations receive no surplus \((u_{t}^{FDC}=0\) for \(t>0\)). An FDC thus replicates the outcome of an IOC: in the case of an IOC, the investor extracts the surplus from all generations of users directly through the access charge; in the case of an FDC, the first generation uses the entry fee to extract the surplus from the second generation, which in turn extracts the surplus from the third one, and so on, so that in effect the first generation extracts the surplus from all future generations, exactly as does the investor through the access charge in the case of an IOC.

2.2.3. Nondiscriminatory cooperative

An NDC that gets off the ground has no trouble to keep going since the new members free ride on the old members’ investment. To be certain, these new members know that their successors will in turn free ride on their own investments, but this cost is discounted. More formally, in steady state, assessments \(a_{t}=a^{NDC}\) satisfy

\[2a^{NDC} = I,\]

so that the net surplus of a date-\(t\) new member \((t>0)\) is equal to

\[u_{t}^{NDC} = u^{NDC} = (1+\delta)\left(\theta - \frac{I}{2}\right),\]

and is positive under condition (1).

A new generation, who controls a reinvestment decision will however choose to keep the NDC going only if

\[(1+\delta)\left(\theta - \frac{I}{2}\right) \geq \theta,\]

which amounts to

\[\delta \theta \geq (1+\delta)\frac{I}{2},\]

and is stronger than condition (1).

The situation is even less favorable for the first generation, so that NDCs do not easily get started. For, that generation bears the brunt of the date-0 investment\(^{12}\) and gets to use the technology only at date 1: \(a_{0}=I, a_{1}=I/2\); generation 0’s utility, if it invests, is therefore\(^{13}\)

\[u_{0}^{NDC} = \delta \left(\theta - \frac{I}{2}\right) - I.\]

And so unless

\[\delta \theta \geq \left(1+\frac{\delta}{2}\right)I,\]

\(^{12}\) Date-0 members may either contribute through lump-sum grants or entry fees, or else commit to an exclusive use of the cooperative at date 0 and pay a surcharge for the use of the (public) technology.

\(^{13}\) Note the importance of the assumption that the NDC cannot exclude. At each instant old members would like to stop investment. They could do so by excluding new entrants or, if the membership were actually declining, by keeping control over the board. More generally, investment in an NDC will take place only if a controlling majority has a forward looking perspective.
the NDC never gets started. Condition (4) is stronger than condition (3) since, unlike the
subsequent ones, the first generation that gets the cooperative going cannot shift half of the
investment burden to the previous generation. This implies that the NDC keeps going if it gets
started.

Therefore, while an FDC performs as an IOC, an NDC appears more fragile. The following
proposition summarizes this analysis:

**Proposition 1.** With homogenous users, an investor-owned corporation and a fully discrim-
inatory cooperative are equivalent and are both viable and efficient. The nondiscriminatory
cooperative is steady-state viable and efficient, but it gets off the ground if and only if condition
(4) holds.

A few remarks are in order. First, the equivalence between investor-owned corporations and
fully discriminatory cooperatives is quite general under user homogeneity and extends for
example to variable usage levels. In effect, the first generation in a fully discriminatory
cooperative owns the facilities and is able to impose monopoly conditions on the new members.

This exercise of monopoly power creates no welfare loss because new members are homogenous
and thus all “get on board”.

Second, we have assumed that discrimination takes the form of an entry fee levied on the
young generation and that this entry fee is used to defray the investment cost. In practice,
discrimination may take several other forms. First, the cooperative may levy seniority-based
assessments. That is, it may levy different access charges \{a^o_i, a^y_i\} on the old and the young.
Second, the association may pay a redemption or dividend \(d_t\) based on capital accounts (that is
here to the old members). Third, the members may be endowed with a transferable property
right. The exiting members then receive a (market determined) lump sum payment \(p_t\) when they
leave the cooperative, which again amounts to reduce their effective usage fee. These various
instruments are formally equivalent in our simple framework: setting a higher charge for young
members \((a^y_i > a^o_i)\) amounts to charging an entry fee \(E_t = a^o_i - a^y_i\); a transferable property right \(p_t\)
reduces the effective fee charged to old members \((a^o_t = a_t - p_t)\); and a redemption \(d_t\) both
reduces the usage fee of the old members \((a^o_t = a_t - d_t)\) and increases that of the young members
\((a^y_t = a_t + d_t)\). We thus have:

**Observation.** A fully discriminatory cooperative can implement its optimal policy by using any
of the following instruments: (i) entry fees; (ii) seniority-based assessments; (iii) redemptions or
dividends; (iv) transferable property rights.

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In broader frameworks, these various instruments would no longer be redundant. For example, an entry fee (paid once
for all) and a redemption policy (paid in several periods) have different impacts on young members’ incentive to stay in
the cooperative in the early stage of their membership. These two instruments also have differentiated impacts on credit
constrained members. There are also two minor caveats to the equivalence demonstrated in the text. The first is that
access charges must be nonnegative; this may be a concern when entry fees are used to reduce the assessment and
variable operating costs are not very high (in the above analysis of the FDC, where operating costs are zero, in steady
state the access charge equals \(a^* = (1 - (1 + \delta) \theta)/(1 - \delta)\) and is indeed negative). Dividends or membership rights avoid this
problem as they allow assessments to remain positive. Conversely assessments should not be so high as to discourage
users. The variety of instruments however allows the cooperative to implement its discrimination policy without striking
these two rocks.
2.3. Robustness and extensions

2.3.1. Network externalities

The analysis above accounts only for cost-sharing network externalities. It is easily generalized to allow also for more conventional network externalities. For example suppose that a member’s gross surplus in a given period \( t \) is given by

\[ \theta + \nu(n_t), \]

where \( n_t \) is the number of date-\( t \) users of the good, and \( \nu \) is an increasing function. The analysis goes through by replacing the per-member benefit \( \theta \) with the “modified benefit” \( \theta = \theta + \nu(2) \). In particular, a for-profit owner or the founders of a fully discriminatory cooperative still appropriate the future flows of network externalities, whereas an NDC finds it more difficult to get started.

2.3.2. Quality of investment

NDCs not only may not make socially desirable investments, but more generally may underscale their investments. The same ideas apply to choices among investments. Consider an inferior investment technology that costs only \( J < I \) but yields a lower surplus \( \gamma \theta \), satisfying

\[ 2\delta(\theta - \gamma) > I - J, \]

so that adopting the superior technology is still the efficient choice.

The investor-owned corporation and the fully discriminatory cooperative fully internalize future benefits and so choose the superior technology in each period. And as before, if a nondiscriminatory cooperative can get started with the new superior technology, the new generations will keep choosing the superior technology as well, since they get half of the benefits but bear only half of the cost. But the founders of an NDC, who bear the full cost of investment and receive only half of the benefits may now choose the inferior technology even if Eq. (4) holds; indeed, if the cooperative keeps investing in the superior technology once it gets started, the first generation gets

\[ U_0^{\text{NDC}} = -I + \delta \left( \theta - \frac{I}{2} \right) \]

if it founds the cooperative with the superior technology, while it gets

\[ \hat{U}_0^{\text{NDC}} = -J + \delta \max \left\{ \gamma - \frac{I}{2}, 0 \right\} \]

if it founds the cooperative with the inferior technology. Therefore, the first generation chooses the inferior technology, even though the superior technology would be viable, whenever

\[ U_0^{\text{NDC}} > U_0^{\text{NDC}} \geq 0, \]

that is, whenever condition (4) holds and, in addition:\(^{15}\)

\[ I - J > \delta(\theta - \gamma). \]

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\(^{15}\) For any \( I \) and \( \theta \) satisfying (4), there always exist \( J \) and \( \gamma \) such that \( 2\delta(\theta - \gamma) > I - J > \delta(\theta - \gamma) \); therefore, the inferior technology may indeed be adopted even when the superior one would be both efficient and viable.
While this short-termist behavior is only transitory here – the cooperative reverts to the superior technology once it gets going\textsuperscript{16} – , it would arise repeatedly if the cooperative were to grow slowly over time, say because of a positive trend in the number of potential users.

2.3.3. Control rights and financing

We assumed that cooperatives did not have access to external finance. External finance raises several issues in the case of a cooperative. Debt finance makes the cooperative highly sensitive to “runs” by members. In the absence of buffer, the desertion by some members increases the assessment levied on remaining members, who then have a strong incentive to leave. Outside equity finance does not create such snowballing. But it (or more generally outside finance) raises control issues. Either outside equityholders have control over the firm and then the latter is run basically as an investor-owned corporation. Or control is granted to the users, in which case outside finance is marred by the “empty-shell syndrome”. In the same way the creditors of a corporation are concerned that shareholders might distribute themselves excessive dividends and empty the firm of its value, outside financiers of a cooperative are preoccupied with the possibility that the profit potential be syphoned off by the members before they can recoup their initial investment. The scope for diversion, though, is different. Dividends received by shareholders are highly visible, so that debtholders can and typically do impose covenants restricting dividend distribution. In contrast, members of a cooperative can distribute themselves less verifiable “dividends” in the form of goldplated versions of the input supplied by the cooperative.

Although our approach compares familiar institutions and is not one of optimal mechanism design, we can shed some further light as to the limits to external financing for cooperatives. To see why the “empty shell” syndrome may discourage external investors from financing cooperatives where members have all the control rights over the assets, consider a simple two-period variant of the model in which: (i) in period 1, users (the cooperative members) can contract with outside investors on current access prices and investment decisions; (ii) in period 2, users decide over second-period access prices. In the second period, the users will set the access price so as to cover operating costs, but have no incentives to generate extra revenue in order to pay back external investors; anticipating this, outside investors will not lend in period 1. This simple illustration suggests that user cooperatives will find it difficult to attract financing without giving some control rights over pricing decisions to the investors. The risk of excessive “frontloading” benefits in earlier periods or of “goldplating” benefits in future periods may similarly discourage external investors.\textsuperscript{17}

3. Competition among organizational forms

Section 2 focused on the viability of alternative organizational forms. Let us now look into the choice of organizational form. The analysis of Section 2 points at two handicaps faced by the nondiscriminatory cooperative form in its competition with alternative institutions. First, it may

\textsuperscript{16} Letting $v \in \{\gamma, \theta\}$ denote the surplus generated by the technology adopted in the previous period, if the date-\textsuperscript{t} generation anticipates that the next one will adopt the superior technology, it gets $v - I/2 + \delta(\theta - I/2)$ if it, too, invests in the superior technology and $\theta - J/2 + \delta \max \{\gamma - I/2, 0\}$ if it invests instead in the inferior technology; thus, even if $\gamma > I/2$ (which is the case when $U_{0 DNC} > U_{0 DNC} > 0$), it prefers the superior technology whenever $2\delta(\theta - \gamma) > I - J$.

\textsuperscript{17} We explored this more formally in Rey and Tirole (2006) (see Appendix D). Using the simple two-period variant just mentioned, we showed that users would indeed have excessive incentives to “frontload” the benefits from investment, in order to reduce repayments to outside investors; similarly, users have incentives to “gold-plate” the investment beyond the efficient level when outside investors cannot fully extract the surplus that users derive from such goldplating.
not be viable. Second, even if it is viable, it is not in general in the interest of the founders to create a nondiscriminatory cooperative: adopting instead a discriminatory charter would allow them to capture some of the future generations’ rents. These two reasons probably explain why most cooperatives actually discriminate.

There are however limits to discrimination. One such limit may come from antitrust enforcement of open access (see Section 5). Another, more along the lines of Section 2, is that a commitment not to discriminate may be necessary to reassure prospective or expanding members. We briefly explore this issue, before turning to the impact of upstream competition.

3.1. Protection against expropriation

Suppose for example that each generation $t$ must incur some fixed investment cost $c$ at date $t-1$ in order to be able to derive gross surplus $\theta$ from access at $t$ and $t+1$. For example, it may make its own technology compatible with that of the platform. If the cooperative can freely discriminate, its incumbent members will expropriate prospective members’ surplus, e.g. by charging higher entry fees to those who have made the required investment (or through any other discriminatory instrument). Anticipating this, prospective members do not incur the fixed cost $c$, and the fully discriminatory cooperative is therefore unable to attract new members.\(^{18}\)

A similar analysis applies to an IOC which cannot commit in advance not to expropriate future users. In contrast, an NDC cannot expropriate future members, who are therefore willing to invest if the fixed cost $c$ is not too large, namely, if $c \leq U^{\text{NDC}}(\theta - \frac{1}{2})$. We thus have:

**Proposition 2.** Suppose that: (i) prospective users must make a relation-specific investment in order to use the technology; and (ii) the technology owner cannot commit to its future access price. Then an IOC or an FDC fails to attract new members, whereas an NDC may still attract new members and be viable if the investment cost is not too large.

Thus, while IOCs and FDCs may face a commitment problem and have problems attracting new members, a non-discrimination charter (or more generally a charter that limits the feasible discrimination) provides a commitment not to expropriate the specific investments made by the future members.\(^{19}\)

In our view, such considerations play an important role in the genesis of cooperatives.\(^{20}\) They may also explain why private property may voluntarily be turned into the public domain. For example, the Visa and MasterCard associations were originally investor-owned.\(^{21}\) Turning the systems into (basically) nondiscriminatory cooperatives enabled the corporation to offer a

\(^{18}\) This issue is similar to the hold-up problem stressed by Williamson (1979) and further studied by Grossman and Hart (1986) and Hart and Moore (1988).

\(^{19}\) Of course, the credibility of such commitment depends in turn on the strength of the charter of the cooperative, and in particular on the conditions under which members may later on amend the non-discrimination provisions of the charter.

\(^{20}\) They may for example contribute to explain the successful creation of agriculture cooperatives in response to the high prices charged by for-profit suppliers. Of course, other, external elements may help cooperatives to get off the ground. For example, nonprofit hospitals in the US have benefitted from charitable contributions. Favorable tax treatment (especially before the Tax Reform Act of 1986) has also contributed to the development of cooperatives. Large R&D consortia often receive seed money or subsidies from governments. For example, European Community Esprit and Eureka IT funding programs in the 1980s contributed respectively to 50% and 35% of the costs of inter-firm R&D projects.

\(^{21}\) Visa finds its origins in Bank of America’s proprietary system, and MasterCard in the merger of proprietary systems owned by Wells Fargo Bank and Marine Midland Bank. MasterCard and Visa recently returned to a for-profit status, even though they are still largely run like associations for the time being.
credible commitment to other issuers and thereby to benefit from increased network externalities. A similar credibility argument often underlies the release of formerly proprietary software to form a coalition around a standard or to initiate an open source process.

3.2. Contestability of upstream segment

Our natural monopoly model has assumed that the platform, regardless of its charter, is not threatened by entry. Let us in contrast assume that the investment technology is widely available, and so a new institution may emerge, that threatens the established platform. We will assume that, when confronted with two alternative offers, the young generation coordinates to jointly take the offer that is best for its users (this is the “Pareto dominance” selection criterion often used in network economics).

Because IOCs do not differ from FDCs under member homogeneity, we will focus on the competition between the FDC and NDC forms. We consider the following timing. At each date t:

(i) If the incumbent organizational form (the one that attracted the young users at date \( t-1 \)) is an FDC, it makes an offer to the young generation. [If the incumbent platform is an NDC, no offer is made since the NDC charter specifies how investment costs are to be shared.]
(ii) There is then free entry into both the NDC and FDC forms: The young generation can decline the incumbent platform’s proposition and either form an NDC or join the FDC that offers the most favorable deal.\(^{22}\)

We will assume that at stage (ii) the young generation chooses the incumbent platform when indifferent with the best of entrant platforms. We also assume that an incumbent platform that does not succeed in attracting the young generation exits; because there is free entry, this assumption is inconsequential. Finally, we will focus on Markov Perfect Equilibria (MPEs): The utility offered to the young generation at stage (i) depends only on the nature of the incumbent platform (NDC or FDC), and the utility offered by entrants is path independent.

Let \( Y^* \) denote the highest (present discounted) utility offered to the young by entrant platforms at stage (ii), and \( O^{*}_{\text{FDC}} \) and \( O^{*}_{\text{NDC}} \) denote the old generation’s equilibrium utility when it has joined an FDC or an NDC in the previous period.

**Lemma 3.** Focusing on entry by FDCs involves no loss of generality.

**Proof.** Note first that \( O^{*}_{\text{FDC}} \geq \theta \), since an FDC can always turn down the next generation by charging a sufficiently high entry price. By contrast, \( O^{*}_{\text{NDC}} \) is equal to either \( \theta \) or \( \theta - I/2 \). And so \( O^{*}_{\text{FDC}} \geq O^{*}_{\text{NDC}} \).

Because the young generation’s initial utility of joining an entrant platform is \(-I\) whether the platform is an entrant NDC or an entrant FDC, the young generation is always better off joining an FDC if it selects an entrant:

\[-I + \delta O^{*}_{\text{FDC}} \geq -I + \delta O^{*}_{\text{NDC}}.\]

\(^{22}\) One can imagine for example that entrepreneurs set up FDCs, invest and demand entry fees. The literature on backward integration studies similar alternatives in static frameworks; see for example Innes and Sexton (1994) and the papers mentioned there.
Let us now look for a stable (i.e., long-lived) organizational form. From the lemma, we know that this stable form must offer to the young:

$$Y^* \geq -I + \delta O^*_{FDC}. \tag{5}$$

### 3.2.1. Fully discriminatory cooperative

If the incumbent platform is an FDC, then its offer is such that Eq. (5) is satisfied with equality:

$$Y^*_{FDC} = -I + \delta O^*_{FDC}.$$  

Because

$$Y^*_{FDC} + O^*_{FDC} = (2\delta - I) + \delta O^*_{FDC},$$

we obtain

$$O^*_{FDC} = 2\delta$$

and

$$Y^*_{FDC} = 2\delta - I \geq 0.$$  

### 3.2.2. Nondiscriminatory cooperative

An NDC is stable if

$$Y^*_{NDC} = (1 + \delta)\left(\theta - \frac{I}{2}\right) \geq -I + \delta O^*_{FDC} = 2\delta - I.$$  

This condition is always strictly satisfied: the NDC is more attractive than an FDC.$^{23}$

We thus conclude that both organizational forms are stable once they get going. However, an FDC wins the initial competition, since, for the date-0 generation:

$$Y^0_{NDC} = -I + \delta \left(\theta - \frac{I}{2}\right) < Y^0_{FDC} = Y^*_{FDC} = -I + 2\delta \theta.$$  

In the case of an FDC all generations of users get the same share of the benefits generated by the cooperative: $Y^0_{FDC} = Y^*_{FDC}$; in contrast, in the case of an NDC the founders get a smaller share of that surplus, at the benefit of future generations of users: $Y^0_{NDC} < Y^*_{NDC}$. This is why an NDC will keep going once it gets started (new generations of users prefer to join an NDC rather than starting a new cooperative), but also why the first generation favors an FDC statute:

**Proposition 4.** Both types of platforms keep going once they get started. However, FDCs are more attractive entrants and thus win the initial competition.

$^{23}$ Furthermore, it is a dominant strategy for the young generation to join the NDC as $(1 + \delta)(\theta - I/2)$ is the minimum it gets by joining the NDC for any continuation outcome.
More generally, FDCs constitute more robust platforms:

### 3.2.3. Shocks

Suppose for example that, in each period, with some positive probability the incumbent’s platform investment cost is very large, so that the platform stops investing (and exits the following period), otherwise the cost is \( I \) as before; and, to fix ideas, suppose that the investment cost is observed by the young before deciding which platform to join. It is easy to check that the most attractive entrants are still FDCs: by joining an entrant, whatever its organizational form, users get \(-I\) in the current period and \( \theta \) in the following period when the cost of investment turns out to be prohibitive. But as old users, when the cost of investment does not turn out to be prohibitive they get again \( 2\theta \) if they previously joined an FDC and only \( \theta - I/2 \) if they joined instead an NDC.\(^{24}\) Therefore, natural selection will lead to an FDC in the long run. A similar argument applies to situations where, with some probability, investing in the incumbent’s platform technology brings little benefit in the following period.

### 3.2.4. Coordination and learning

Coordination failures (for example, with some probability, the young generation does not coordinate in the current period on what is optimal for it) or learning costs may also favor the emergence of FDCs. If for example the young generation must incur a learning cost \( L \), the benefit from joining an NDC becomes

\[
Y_{\text{NDC}}^* = (1 + \delta) \left( \theta - \frac{I}{2} \right) - L.
\]

In contrast, in the case of an (incumbent or entrant) FDC, the learning costs are partly absorbed by the old generation (or by the entrepreneur starting the entrant FDC): as before, the young generation gets

\[
Y_{\text{FDC}}^* = -I + \delta O_{\text{FDC}}^*,
\]

and since now

\[
Y_{\text{FDC}}^* + O_{\text{FDC}}^* = (2\theta - L - I) + \delta O_{\text{FDC}}^*,
\]

we obtain

\[
O_{\text{FDC}}^* = 2\theta - L
\]

and

\[
Y_{\text{FDC}}^* = 2\delta \theta - I - \delta L.
\]

\(^{24}\) Denoting by \( \lambda \) the probability of a bad shock on the cost of investment, and by \( Y_{\text{FDC}} \) and \( O_{\text{FDC}} \) the expected present discounted utility of the young and the utility of the old when the cost of investment stays equal to \( I \), condition (5) becomes

\[
Y_{\text{FDC}} \geq -I + \delta[(1 - \lambda)O_{\text{FDC}} + \lambda \theta],
\]

leading to

\[
Y_{\text{FDC}}^* = -I + \delta \left[ (1 - \lambda)O_{\text{FDC}}^* + \lambda \theta \right],
\]

while by construction

\[
Y_{\text{FDC}}^* + O_{\text{FDC}}^* = (2\theta - I) + \delta \left[ (1 - \lambda)O_{\text{FDC}} + \lambda \theta \right].
\]

Thus \( O_{\text{FDC}}^* = 2\theta \) and

\[
Y_{\text{FDC}}^0 = -I + \delta [2\theta + (1 - \lambda)2\theta] Y_{\text{NDC}}^0 = -I + \delta \left[ 2\theta + (1 - \lambda) \left( \theta - \frac{I}{2} \right) \right].
\]
In other words, while the young generation fully bears the costs of coordination or learning in the case of an NDC (its utility is reduced by \( L \)), in the case of an FDC these costs are shared with the old generation (the utility of the young generation is reduced by \( \delta L \) only), which can tilt the balance in favor of FDCs.

**Remark.** This fragility of NDCs to entry by FDCs may remind the reader of the political economy of pensions. Pay-as-you-go systems and FDCs are less favorable to the young (as opposed to the old) than fully-funded-social-security systems and NDCs. As is well-known it is quite difficult to move from a pay-as-you-go system to a fully-funded system while the reverse is obviously easier.

### 4. Heterogeneous users and inclusiveness

#### 4.1. Dynamics of membership

Allowing users to enjoy different benefits is interesting for two reasons. First, heterogeneity introduces a distinction between viability and efficiency: an organizational form may support investment, but be underinclusive and therefore inefficient; in contrast, with homogenous users, viability always implied efficiency. Second, heterogeneity creates some differentiation between the investor-owned corporation and the discriminatory cooperative.

Let us assume that, in a given generation, the agents’ gross surpluses are distributed according to cumulative distribution \( F(\theta) \) on \([0, \infty)\). We assume that the distribution is log concave:

\[
\frac{1}{1 - F(\theta)} \frac{f(\theta)}{F(\theta)} \quad \text{increases with} \quad \theta.
\]

**4.1.1. Investor-owned corporation**

Let

\[
\theta^m = \arg \max \{\theta[1 - F(\theta)]\}
\]

denote the “monopoly cut-off”, that is the type of the marginal user of the technology when the technology is marketed by an IOC.\(^{25}\) The monopoly profit is

\[
\nu^m = \frac{2\delta \theta^m[1 - F(\theta^m)] - I}{1 - \delta} = \frac{\delta \pi^m - I}{1 - \delta}.
\]

We assume that an IOC is viable:

\[
\delta \pi^m > I.
\]

\(^{25}\) We assume that the IOC cannot discriminate among the users according to their age. Otherwise, the IOC would offer nonmember access at a low price, targeted to old users who have not joined in the previous period. From standard intertemporal (Coasian) price discrimination theory, we know that this would discourage some users (those with type only slightly above \( \theta^m \)) from subscribing when they are young, which would lower the profitability of the IOC: A policy of no discrimination between short- and long-term users allows the IOC to commit to monopoly access charges. We will make the similar assumption for the NDC and the FDC. Note in particular that this assumption understates the extent of free riding in cooperatives as it eliminates “footdragging” (the strategy adopted by some potential members of waiting until investment has been sunk to adhere to the cooperative).
4.1.2. Fully discriminatory cooperative

In each period \( t \), given the number of old members \( 1 - F(\theta_{t-1}) \), the entry fee \( E_t \) and the charge \( a_{t+1} \) anticipated for the next period, users with a high enough \( \theta (\theta \geq \theta_t \), say) join the FDC while those with a lower \( \theta \) do not; conversely, in the next period, the access charge \( a_{t+1} \) will be determined by the optimal entry policy set by the young generation, and thus depends on the threshold \( \theta_t \); this threshold and the corresponding access charge \( a_t \) are thus such that:

\[
a_t + E_t = (1 + \delta)\theta_t - \delta a_{t+1}(\theta_t),
\]

where \( a_{t+1}(\theta_t) \) denotes the access fee generated by the young generation in the next period, and

\[
I = [1 - F(\theta_{t-1})]a_t + [1 - F(\theta_t)](a_t + E_t).
\]

The old members have unanimous preferences over the entry fee \( E_t \) and seek to solve

\[
\min_{\{\theta_t, a_t, E_t\}} a_t,
\]

subject to Eqs. (7) and (8). Clearly, the value of the program, \( a_t (\theta_{t-1}) \), satisfies

\[
[1 - F(\theta_{t-1})]a_t(\theta_{t-1}) = \min_{\theta_t} \{I - [1 - F(\theta_t)][(1 + \delta)\theta_t - \delta a_{t+1}(\theta_t)]\}
\]

\[
= I - \max_{\theta_t} \{(1 + \delta)[1 - F(\theta_t)]\theta_t - \delta [1 - F(\theta_t)]a_{t+1}(\theta_t)\}
\]

\[
= R - (1 + \delta) \max_{\theta_t} \{[1 - F(\theta_t)]\theta_t\},
\]

where

\[
R = (1 + \delta)I - \delta \max_{\theta_{t+1}} \{[1 - F(\theta_{t+1})][(1 + \delta)\theta_{t+1} - \delta a_{t+2}(\theta_{t+1})]\}
\]

does not depend on \( \theta_t \). Generation \( t-1 \) will thus choose \( \theta_t \) so as to maximize \( [1 - F(\theta_t)]\theta_t \), which leads to: \(26\)

\[
\theta_t = \theta^m.
\]

Except in the first period, the membership is the same as for an IOC and users with type \( \theta > \theta^m \) get positive surplus \( (1 + \delta)(\theta - \theta^m) \). The initial membership is however wider than with an IOC,

\[\text{This is achieved by setting a fixed fee } E^m \text{ and an access charge } a^m \text{ such that } a^m + E^m = (1 + \delta)\theta^m - \delta a^m \text{ and } (2a^m + E^m) [1 - F(\theta^m)] = I, \text{ implying that}
\]

\[
E^m = \frac{1 + \delta}{1 - \delta} \left[ 2\theta^m - \frac{I}{1 - F(\theta^m)} \right],
\]

\[
a^m = \frac{1}{1 - \delta} \left[ \frac{I}{1 - F(\theta^m)} - (1 + \delta)\theta^m \right].
\]
since initial members distribute among themselves the surplus generated by the cooperative \((\theta_0^{\text{FDC}} < \theta_m^{\text{m}})\).

### 4.1.3. Nondiscriminatory cooperative

In the steady state of an NDC, the marginal user’s type \(\theta^{\text{NDC}}\) is equal to the steady-state access price \(a^{\text{NDC}}\). And so \(\theta^{\text{NDC}}\) is given by (the smallest root of):

\[
2\theta^{\text{NDC}}[1 - F(\theta^{\text{NDC}})] = I.
\]

The IOC is underinclusive relative to the NDC, since \(h^{\text{NDC}} < h^{\text{m}}\).

While the steady-state outcome under an NDC is socially superior to the IOC outcome, there may be serious transition problems like in the case of homogenous users. The first generation bears the initial cost of investment but does not immediately benefit from the joint venture. So the NDC may never get off the ground. It can get going at date 0 only if there exists a sequence of marginal customers \(\theta_0, \theta_1, \ldots, \theta_t, \ldots\), and uniform access charges \(a_0, a_1, \ldots, a_t, \ldots\), such that

\[
a_0[1 - F(\theta_0)] = I,
\]

\[
a_t[1 - F(\theta_{t-1}) + 1 - F(\theta_t)] = I \quad \text{for all } t \geq 1,
\]

and

\[
\delta \theta_0 = a_0 + \delta a_1,
\]

\[
(1 + \delta)\theta_t = a_t + \delta a_{t+1} \quad \text{for all } t \geq 1.
\]

The interpretation is as follows. If at date 0, agents with type \(\theta \geq \theta_0\) “contribute” (pay \(a_0\)), they get nothing in period 0 but the venture gets started, and so they will be able to benefit from the investment in period 1, provided they pay the access price \(a_1\). And so on.

We will say that a sequence \(\Theta = (\theta_0, \theta_1, \ldots)\) is self-financing if for all \(t \geq 1\)

\[
(1 + \delta)\theta_t \geq \left[\frac{1}{1 - F(\theta_{t-1}) + 1 - F(\theta_t)} + \frac{\delta}{1 - F(\theta_t) + 1 - F(\theta_{t+1})}\right]I,
\]

and

\[
\delta \theta_0 \geq \left[\frac{1}{1 - F(\theta_0)} + \frac{\delta}{1 - F(\theta_0) + 1 - F(\theta_t)}\right].
\]

---

\(\theta_0^{\text{FDC}}\) is determined by

\[
\delta \theta_0 [1 - F(\theta_0)] = I + \delta[I - [1 - F(\theta_0)][E - a^m]]
\]

which leads to:

\[
\delta \theta_0 [1 - F(\theta_0)] - \theta_0 [1 - F(\theta_0)] = V^m,
\]

and thus \(\theta_0^{\text{FDC}} < \theta_m^{\text{m}}\) whenever \(V^m > 0\).
That is, if at date $t$ type $\theta_i$ is willing to join the venture provided all types above $\theta_i$ also join, and at dates $t-1$ and $t+1$ types above $\theta_{t-1}$ and $\theta_{t+1}$, respectively, have joined and will join the venture. If it is nonempty, the set of types $\theta_i$ satisfying this condition for given $\theta_{t-1}$ and $\theta_{t+1}$, has a lowest element, and this lowest element is nondecreasing in $\theta_{t-1}$ and $\theta_{t+1}$. In words, potential users are willing to become members at date $t$ if the venture is already bigger ($\theta_{t-1}$ smaller), since a wide membership spreads the fixed cost over a larger number of members ($a_t=1/[1-F(\theta_{t-1})]+1-F(\theta_t)$) and if it is expected that more users will join later on ($\theta_{t+1}$ smaller), since it reduces the anticipated level of assessments for the next period. Appendix A shows that if sequences $\Theta=(\theta_0, \theta_1, \ldots)$ and $\Theta'=(\theta_0', \theta_1', \ldots)$ are self-financing, so is $\Theta \cap \Theta'=(\min(\theta_0, \theta_0'), \min(\theta_1, \theta_1'), \ldots)$:

Lemma 5. If a self-financing sequence exists, then there exists a smallest self-financing sequence, $\hat{\Theta}=(\hat{\theta}_0, \hat{\theta}_1, \ldots)$ which thus Pareto-dominates (from the point of view of all generations) all other self-financing sequences. This smallest sequence decreases over time ($\hat{\theta}_{t+1}<\hat{\theta}_t$) and converges towards $\hat{\theta}_\infty=\theta^{NDC}$.

A self-financing sequence may not exist. In particular, the above conditions imply

$$\delta(\theta_0-a_1)[1-F(\theta_0)]=I.$$ 

Since $a_1>0$, no such sequence exists, even though the NDC is steady-state viable, if for example

$$\frac{I}{2\delta}<\theta^m[1-F(\theta^m)]<\frac{I}{\delta}.$$ 

Proposition 6. Under user heterogeneity, the investor-owned corporation and the discriminatory cooperative are viable if and only if condition (6) holds. They are then equally inclusive except at the initial stage where the cooperative is more inclusive. The nondiscriminatory cooperative is more inclusive than the other two organizational forms, but is viable under a condition that is stricter than Eq. (6).

4.2. Social optimality

Under heterogeneity, the FDC yields the monopoly membership level. On the other hand, the NDC may not be viable. Let us now consider the Ramsey optimum, defined as the allocation that maximizes the present discounted user surplus:

$$\max_{\{\theta_0, \theta_1, \ldots\}} \int_{\theta_0}^{\infty} \delta(\theta-\theta_0)f(\theta)d\theta + \sum_{t=0}^{\infty} \delta^t \left[ \int_{\theta_t}^{\infty} (1+\delta)(\theta-\theta_t)f(\theta)d\theta \right]$$ 

s.t. $\delta\theta_0[1-F(\theta_0)] - I + \sum_{t=0}^{\infty} \delta^t [(1+\delta)\theta_t[1-F(\theta_t)] - I] \geq 0$.

The maximand reflects the fact that for each generation the net surplus of the marginal user $\theta_t$ is equal to zero and therefore the rent of type $\theta$ is $(1+\delta)(\theta-\theta_t)$ for $t \geq 1$ and $\delta(\theta-\theta_0)$ for $t=0$. The budget constraint accounts for the equality between the marginal type’s gross surplus, $(1+\delta)\theta_t$, and his net intertemporal payment to the cooperative. Unsurprisingly the Ramsey optimum in this stationary context is a constant cutoff, $\theta_r=\theta^R$; each type $\theta>\theta^R$ then gets rent $(1+\delta)(\theta-\theta^R)$ (or $\delta(\theta-\theta^R)$ in the first generation). From the budget constraint, the cutoff $\theta^R$ is the smallest root of

$$2\delta\theta^R[1-F(\theta^R)]=I.$$
The following proposition is proved in Appendix B:

**Proposition 7.**
(i) The Ramsey optimal cooperative has a constant membership ($\theta_t = \theta_R$ for all $t$).
(ii) It is more inclusive than the fully discriminatory cooperative and the investor-owned corporation but less inclusive than a steady-state nondiscriminatory cooperative (assuming the latter can get off the ground).

4.2.1. Comparison with public utility regulation

The Ramsey allocation can be achieved by a leveraged public utility. Two preliminary remarks are in order. First, we will adopt an idealized ("Ramsey–Boiteux") perspective on public utilities; we deliberately ignore the inefficiencies attached to this form of regulation and only aim at a better conceptual understanding of the result obtained above. Second, we have argued that cooperatives have little or no access to external financing because users can easily pay themselves dividends in kind. Public utilities are (highly) leveraged consumer cooperatives. The difference is that public utilities are subject to intensive regulation and to the legal obligation, enforced by courts, to provide investors with a fair rate of return.

Consider thus a regulated NDC with access to debt financing. Leverage allows the NDC to get off the ground by spreading the initial cost across generations. Suppose for example that the cooperative is allowed to impute a fair rate of return ($1/\delta - 1$) on nondepreciated investment to the current access price. That is, since investments are fully depreciated after two periods, the NDC sets $a_0=0$ and, at each date $t \geq 1$, an access charge $a_t$ satisfying

$$a_t[1 - F(\theta_{t-1}) + 1 - F(\theta_t)] = \frac{I}{\delta}.$$

This rate-of-return regulation allows the venture to lever on a permanent basis (borrow $I$ at each date, and reimburse $I/\delta$ at the following date). This policy corresponds to the celebrated "golden rule" that Keynes and Pigou designed as a constraint on leverage for country or municipality indebtedness, according to which only capital, and no current, expenditures could be financed through debt.

From a financial viewpoint, everything is as if the investment were sunk at the date at which it bears fruits; and so the outcome is then the Ramsey steady-state outcome, characterized by $\theta_t = \theta_R$ for every $t \geq 0$ and $a_t = a^R = \theta_R$ for $t \geq 1$.

5. Downstream competition and foreclosure

We have so far assumed that new members do not reduce the value of membership for existing members. This is no longer so if they compete on the same product market. One may therefore wonder whether imposing open access gives rise to a "deregulatory taking." In the case of cooperatives, though, there are stricto sensu no shareholders whose investments in an essential input are expropriated through the increase in competition. Hence, a simple-minded analogy is not warranted and we must conduct a separate analysis.

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28 Alternatively, the Ramsey optimum can be achieved by setting an entry fee $E_t = 2(1 + \delta)\theta^R (E_0 = 2\delta \theta^R$ for the first generation) and by subsidizing usage: $a_t = -\theta^R$. Such a subsidy may trigger moral hazard problems when the input can be used in variable proportions. In order to avoid these problems and, more generally, to avoid usage distortions, a two-part usage tariff should be used — the subsidy should then be applied to the fixed part.
We assume now that investing \( I \) in any given period leads to the development of a new technological generation in the following period. More precisely, and to keep the analysis simple, we will assume that in each period \( t \):

- previous generations of the technology are freely available, regardless of the history of the industry;
- by contrast, generation-\( t \) of the technology is available only if investment \( I \) was sunk in period \( t-1 \).

We will assume that competition among the users of the previous generations of the technology dissipate their profits. In contrast, the latest generation generates additional surplus and positive profits, which we assume to be stationary over time.\(^{29}\) We will assume that increasing the number \( n \) of users of the latest generation decreases the per-firm profit, \( \pi(n) \), whereas the aggregate profit, \( \Pi(n) = n\pi(n) \), is concave and maximal for a finite number of users, \( n^\Pi \), and total welfare, \( W(n) \), is quasi-concave and such that \( W'(n) > 0 \) whenever \( \Pi'(n) \geq 0 \). The latter assumption reflects competition authorities’ traditional concern about cooperatives being underinclusive.\(^{30}\)

There is an infinite number of potential users; for simplicity we will treat the number \( n \) of actual users as a continuous variable, but nothing hinges on this. Also, for expositional simplicity we will now assume that users are infinitely lived, although the analysis would be the same if they were two-period lived as in the rest of the paper. We will suppose that the investment is viable:

\[
\delta \Pi(n^\Pi) > I, \tag{9}
\]

but that its duplication is not viable:\(^{31}\)

\[
\max_n \delta[\Pi(n^\Pi + n) - \Pi(n^\Pi)] < I. \tag{10}
\]

We will moreover assume that the developers of the latest technological generation have an (arbitrarily small) timing advantage for developing the next generation; thus, if a corporation or a

---

\(^{29}\) This supposes some separability in consumers’ preferences. Consider for example a standard discrete choice model where: (i) each household \( h \) is willing to buy one unit and obtain a net surplus

\[
u_t + \theta^t_i - p_i
\]

when buying from a firm \( i \) that has access to generation \( t \); (ii) firms produce at no cost, say, whereas the \( \theta^t_i \) are drawn independently from a continuous distribution over \((-\infty, 0]\). At date \( t \), perfect competition among the users of the previous generations yields a net surplus (gross of \( \theta \)) \( u_{t-1} \). Competition among the \( n \) firms that have access to the up-to-date technology thus generates profits and welfare increase that depend only on \( n \) and \( u_t - u_{t-1} \). If we further assume that \( u_t - u_{t-1} \) remain constant over time, then \( \pi(n) \) and \( W(n) \) are also time-independent.

\(^{30}\) See e.g. Rey and Tirole (in press) for a survey of the recent literature and a general discussion on vertical foreclosure.

\(^{31}\) Given the concavity of \( \Pi(\cdot) \), condition (10) implies that duplicating the investment is a fortiori not profitable when more than \( n^\Pi \) firms are already using the new technology, since:

\[
\frac{\partial}{\partial n}[\Pi(n + m) - \Pi(n)] = \Pi'(n + m) - \Pi'(n)
\]

\[
= \int_n^{n+m} \Pi''(x)dx < 0.
\]
cooperative has developed the generation-$t$ technology at date $t$, then if it wishes so it can preempt and thus discourage any other potential developer from investing in the next generation.

Membership fees take the form of a fixed payment $a$, and thus have no direct impact on prices, welfare and (gross) profits. This rules out the potential use of membership fees as a “collusive device” designed to maintain prices above the competitive level.\(^{32}\)

As earlier, we will analyze the following institutions:

*Investor-owned corporation* (IOC) If it invests in period $t$, the IOC owns the generation-$t$ technology at date $t$ and licenses it at access charge $a_t$.

*Nondiscriminatory cooperative* (NDC) If the NDC has invested in the previous period, it owns the generation-$t$ technology but any firm can join (or stay in) the cooperative and use that technology. In each period, current members first decide whether to invest in the next generation of the technology, and set the access charge $a_t$ so as to cover the costs; then, they decide whether to stay and any other firm can join the NDC and, for the same amount $a_t$, use the generation-$t$ technology.

*Fully discriminatory cooperative* (FDC) As before, we will formalize discrimination as the existence of an entry fee $E_t$ chosen by established members and to be paid by new members, in addition to the access charge $a_t$. The governance is the same as for an NDC, except that current members set the entry fee as well as the access charge.

Before analyzing these institutions, it is useful to identify the social optimum.

5.1. Social optimum

In a first-best world, the technology should be made available to as many users as possible; however, this would dissipate profits and would not allow technology developers to recoup their investment. Absent any subsidy from third parties, the second-best number of users should therefore maximize the discounted sum of welfare flows, subject to a budget constraint:

$$\max_{\{n_t\}_{t=1}} \sum_{t \geq 0} \delta^t [\delta W(n_t) - I]$$

s.t. $\sum_{t \geq 0} \delta^t [\delta \Pi(n_t) c - I] \geq 0$.

This optimal number of users is then stationary\(^{33}\) and thus equal to the Ramsey value, $n^R$, which is the (largest) solution to:

$$\delta \Pi(n^R) = I.$$

\(^{32}\) Members could otherwise maintain high prices (even at the monopoly level) by inflating the usage price charged for the input supplied by the cooperative and sharing the proceeds through dividends, lower entry or franchise fees, in-kind benefits, etc. Competition authorities are of course aware of this possibility and may intervene. More generally, we assume that the cooperative is not a front for a cartel suppressing competition. There is a wide consensus in the law and economics literature (Baker (1993), Carlton and Salop (1996), Chang et al. (1998)) that joint ventures should not facilitate collusion, implement naked price fixing, limit output, prevent offerings of new products, exclude low-cost competitors, and so forth.

\(^{33}\) Letting $\mu$ denote the Lagrangian multiplier associated with the budget constraint, the first-order condition yields

$$W'(n_t) + \mu \Pi'(n_t),$$

which implies that $n_t$ is constant over time.
In particular, it is here desirable to allow more users than what is privately optimal: the viability condition (9) implies \( n^R > n^\Pi \).

5.2. Private investment

We now study the various organizational forms’ incentives to invest. Intuitively, open access policy can discourage investment by depriving investors from an appropriate return. Conversely, closed access policies encourage investment by protecting these returns, but they may excessively restrict access, compared with what would be socially desirable.

5.2.1. Investor-owned corporation

An IOC can choose the number of users and extract their profits through e.g. an entry fee. It thus solves:

\[
\max_{\{n_t\}_{t=1}} \sum_{t \geq 0} \delta^t [\delta \Pi(n_t) - 1],
\]

which leads to

\[ n^{\text{IOC}} = n^\Pi. \]

That is, the IOC excessively limits access to the technology in order to avoid profit dissipation.

5.2.2. Fully discriminatory cooperative

The founders of an FDC can again replicate the outcome of an IOC: by setting the entry fee at a prohibitive level, they can discourage any further entry and thus implement a “closed access” policy which shields them from additional competitive pressure. More precisely, by forming an FDC and charging any entry fee higher than \( E^\Pi = \pi(n^\Pi) - I/n^\Pi \), \( n^{\text{FDC}} = n^\Pi \) founding members can secure and share among themselves the maximal industry profit; the access fee \( a_t \) can then be used to share the cost of the investment among them: \( a_t = a^\text{FDC} = I/n^\Pi \).

5.2.3. Nondiscriminatory cooperative

Consider now the case of an NDC. If the cooperative invested at date \( t-1 \), then at date \( t \) users join or stay in the cooperative as long as their profit, \( \pi(\cdot) \), exceeds the access charge \( a_t \). Therefore, the total number of members, \( n_t \), satisfies:

\[ \pi(n_t) = a_t, \]

and thus the cooperative’s members will make zero profit. But then, at date \( t-1 \) the cooperative’s members, anticipating that their will obtain no profit in the following period, whether they stay in the cooperative or not, do not invest.

We thus have:

**Proposition 8.**

(i) The “open access” characteristic of NDCs discourages their members from investing in the technology.

(ii) IOCs and FDCs both result in an “closed access” policy that encourages investment but excessively restricts access to the technology.

(iii) Total welfare is higher for IOCs and FDCs than for NDCs, and is suboptimal.
5.3. Access holidays, leverage and the golden rule

In order to get closer to the social optimum, regulators can either constrain the exercise of market power by IOCs and FDCs, or allow some investment protection in the case of NDCs. Consider first the case of an FDC (the analysis would be similar for an IOC). In the absence of any regulatory intervention, the cooperative would invest in the technology but would restrict access beyond what is needed to recoup the investment; potential solutions to this problem thus involve some form of mandatory access, e.g. by imposing additional licenses or by putting a cap on the entry fee. Doing so amounts to regulating directly or indirectly the return on investment, and thus involves the usual issues (access to the relevant information on costs and benefits, commitment problems, and so forth) attached to such “heavy-handed” regulation.

Consider next the case of an NDC. Absent any remedy, there would be excessive access if the investment was made, which in turn deters investment. Remedies must therefore at least partially protect the incumbent members from competitive pressure. A first series of measures consist in introducing a dose of discrimination, e.g. through limited entry fees or any other mechanisms that discriminates between new users and incumbent ones. This, in effect, makes the cooperative more similar to an FDC.

5.3.1. Leverage

One possibility consists in allowing some leverage. Suppose for example that, in each period, the cooperative is allowed to borrow up to a maximal debt level $D\bar{\delta}$ to be repaid in the following period. At date $t$, given the debt $D_{t-1}$ contracted in the previous period and the current number of members $n_t$, contracting debt $D_t$ leads to an access charge $a_t$ such that:

$$n_t a_t = I + \frac{D_{t-1}}{\delta} - D_t.$$

The cooperative thus always borrows as much as possible (i.e., $D_t = D\bar{\delta}$), in order to reduce the current access price and pass the burden on to future members.

As before, at each date the cooperative members anticipate that, due to free entry, they will make zero profit in the following period whether they stay in the cooperative or not. Therefore, in the initial period, in which no technological advantage can be obtained, the access fee must be non-positive, which in turn implies that the debt level must cover the full cost of the investment: $D \geq I$. Conversely, allowing the NDC to be fully leveraged (this is the golden rule already discussed in Section 4.2) enables its founding members to break even. Indeed, if the cooperative systematically borrows $D = I$ to cover the investment cost, in the starting period it does not need to levy any access fee and in each of the following periods it will need to levy

$$I + \frac{D}{\delta} - D = I - \frac{I}{\delta} = \Pi(n^R);$$

it will thus set the access fee to $\pi(n^R) = \Pi(n^R)/n^R$ and attract $n^R$ users.

More generally, borrowing $\bar{D}$ leads to an access fee such that:

$$\delta n a = \delta n \pi(n) = \delta \Pi(n) = \delta I + (1 - \delta) \bar{D}.$$

---

34 While we focus here on yes/no investment decisions, more generally the regulated return should provide adequate incentives to invest to the desirable level.
Therefore, allowing the cooperative to be more than fully leveraged would lead to too few users: this would yield \( \delta \Pi(n) > I \), and thus \( n < n^R \). It is therefore optimal to allow the NDC to borrow up to, but no more than the amount of its investment.\(^{35}\)

### 5.3.2. Access holiday

Another form of protection consists in granting limited “access holidays”. Suppose for example that investment is lumpy: it takes place at the beginning of a “period”, which is subdivided into subperiods (an infinite number for expositional simplicity). Suppose further that \( n \) founding members are allowed to deny access during a fraction \( \alpha \) of each period, while during the remaining fraction \( 1 - \alpha \) of the period, any additional user can pay the fixed access charge \( a \) and enjoy the profit \( \pi(\cdot) \). If the cooperative gets started, in each of the following periods the access charge \( a \) (during and outside access holidays) and the number of entrants, \( m(\alpha) \), then satisfy

\[
\pi(n + m) = a,
\]

and

\[
[\alpha n + (1 - \alpha)(n + m)]a = I,
\]

and thus:

\[
[n + (1 - \alpha)m] \pi(n + m) = I.
\]

The limit case \( \alpha = 1 \) corresponds to closed access and thus replicates the outcome of an IOC or FDC. Conversely, the limit case \( \alpha = 0 \) corresponds to open access and does not allow to recoup members the investment cost; the length of the access holiday should thus be at least sufficient to allow the founding members to recoup their investment. In addition, limiting the duration of the access holiday (i.e., reducing \( \alpha \)) reduces the number of additional users. Indeed, letting \( \varphi(m, \alpha) \) denote the left-hand side of the above equation, we have:

\[
\frac{\partial \varphi}{\partial \alpha} = -m \pi(n + m) < 0,
\]

and

\[
\frac{\partial \varphi}{\partial m} = [n + (1 - \alpha)m] \pi'(n + m) + (1 - \alpha) \pi(n + m)
\]

\[
= (1 - \alpha)\Pi'(n + m) n \pi'(n + m),
\]

where the second term is negative and the first term is also negative in the relevant range (i.e., for \( n + m > n^{II} \)); therefore:

\[
m'(\alpha) = -\frac{\partial \varphi}{\partial \varphi} < 0.
\]

\(^{35}\) It might be socially desirable to restrict the number of users below the Ramsey level when there are significant economies of scale or scope (e.g., large set-up costs): this would arise here if \( \Pi(n) \) were maximal for a finite \( n^w \), satisfying \( \Pi(n^w) > I / \delta \). It would then be desirable to allow the NDC to be more than fully leveraged; the initial access fee would then be negative (that is, lower than the operating cost), allowing in effect the founding members to cover their investment cost through the profit generated by future use. However, as for return regulation, assessing the desirable amount of leverage (which is equal to \( \delta(\Pi(n^w) - I) / (1 - \delta) \)) then requires detailed information on costs and benefits as well as on the discount rate.

The optimal policy thus also involves a trade-off between granting access to more users, or for a longer period of time. Let us note, though, that access holidays:

- require much more information than the golden rule,
- imply a non-stationary level of market power, and therefore do not implement the socially optimal policy.

6. Conclusion

Potential members knocking at the door of a successful joint venture always feel slighted when offered discriminatory treatment or being excluded altogether. This paper has analyzed their concern and identified two potential sources of inefficiency arising from discriminatory treatment. The first is that, in a natural monopoly situation, incumbent members have an incentive to exploit their monopoly power; the resulting taxation of new membership leads to underinclusiveness (Section 4). The second source of inefficiency is due to the incumbents’ incentive to restrict entry into their downstream markets by new players (Section 5).

This suggests that, in natural monopoly situations, joint ventures ought to be viewed as “essential facilities” and forced to treat users equally. Our analysis however calls for some caution, at least at a general level. Nondiscriminatory cooperatives are highly fragile institutions. For one thing, they imply that new members free ride on the investment of established members (had we introduced uncertainty, free riding might have been even more of an issue as potential members could join the joint venture only if it turns successful). This induces underinvestment (the horizon problem) or even prevents the cooperative from getting off the ground. Furthermore, even if it is viable on a stand-alone basis, the nondiscriminatory cooperative is vulnerable to attacks by discriminatory cooperatives or by for-profits, which can lure potential members through the promise of future natural monopoly profits. For another thing, in a situation in which new members compete with established ones on the product market, the nondiscriminatory cooperative may be reluctant to levy assessments that reduce the latter’s current profit in order to finance an innovation whose benefits will be competed away. While the new members’ concerns are real, these aspects should be seriously taken into account before forcing open access to a joint venture. In a nutshell, open access policies involve a familiar Schumpeterian trade-off between static efficiency and innovation. Last, future research should aim at helping policymakers to define an “organizationally neutral” competition policy. The treatment of access to cooperatives should be consistent with the essential facilities doctrine applied to investor-owned corporations, and not tilt the level-playing field in favor of a specific organizational form.

36 If returns to scale are moderate beyond some minimal scale, facilities duplication may substitute favorably for a necessarily imperfect regulation of access. This point was for example made forcefully by Advocate General Jacobs in Oscar Bronner vs Mediaprint (European Court of Justice 1998). Mediaprint (with downstream market share of 47%) operated its own newspaper delivery system in Austria, and refused to give access to a competing newspaper, Der Standard (market share 3.6%), on the same terms as a noncompeting, independent newspaper that used the delivery system. The Advocate General expressed his concerns that an access policy, while encouraging competition in the short run, would kill incentives for small newspapers to develop their own delivery system (possibly cooperatively) and thereby prevent facilities-based competition in the long term.

37 The organizational neutrality problem has been recognized at least since Associated Press (Associated Press II, 326 US 1943). The US Supreme Court affirmed a lower court decision and sided with the government challenge of AP’s bylaws. Dissenting Justices however noted that AP’s two proprietary competitors, United Press and International News Service, were able to enforce unchallenged similar restraints as those implied by AP’s bylaws in their contracts with subscribers.
Appendix A. Proof of Lemma 5

Consider two self-financing sequences $\Theta^1=(\theta^1_0, \theta^1_1, \ldots)$ and $\Theta^2=(\theta^2_0, \theta^2_1, \ldots)$; by definition, they must satisfy, for $i=1, 2$

$$
\delta \theta^i_0 = \left[ \frac{1}{1 - F(\theta^i_0)} + \frac{\delta}{1 - F(\theta^i_0) + 1 - F(\theta^i_1)} \right] \quad (C_0^i)
$$

and for $t=1, 2, \ldots$

$$
(1 + \delta) \theta^i_t \geq \left[ \frac{1}{1 - F(\theta^i_{t-1}) + 1 - F(\theta^i_t)} + \frac{\delta}{1 - F(\theta^i_t) + 1 - F(\theta^i_{t+1})} \right] I. \quad (C_t^i)
$$

Now, for each $t=0, 1, \ldots$, choose $i(t)$ such that $\theta^{i(t)}_t = \min(\theta^1_t, \theta^2_t)$. Then for any $t>0$, $(C_t^{i(t)})$ implies

$$
(1 + \delta) \min(\theta^1_t, \theta^2_t) = (1 + \delta) \theta^{i(t)}_t \geq \left[ \frac{1}{1 - F(\theta^{i(t)}_{t-1}) + 1 - F(\theta^{i(t)}_t)} + \frac{\delta}{1 - F(\theta^{i(t)}_t) + 1 - F(\theta^{i(t)}_{t+1})} \right] I
$$

$$
\geq \frac{1 - F(\min(\theta^1_{t-1}, \theta^2_{t-1}), \theta^1_t)}{1 - F(\min(\theta^1_{t-1}, \theta^2_{t-1})) + 1 - F(\min(\theta^1_t, \theta^2_t))}
$$

$$
\delta I + \frac{1 - F(\min(\theta^1_t, \theta^2_t)) + 1 - F(\min(\theta^1_{t+1}, \theta^2_{t+1}))}{\delta I},
$$

(and similarly for $t=0$), which establishes that the sequence $\Theta^1 \land \Theta^2=(\min(\theta^1_0, \theta^2_0), \min(\theta^1_1, \theta^2_1), \ldots)$ is self-financing. This, in turn, ensures that if there exists a self-financing sequence, there exists a smallest one, which we denote by $\hat{\Theta}=(\hat{\theta}_0, \hat{\theta}_1, \ldots)$. Furthermore, this smallest self-financing sequence must be such that all constraints are binding (otherwise, it would be possible to reduce the first $\hat{\theta}_t$, say, for which the corresponding constraint is not binding):

$$
\delta \theta_0 = \left[ \frac{1}{1 - F(\theta_0)} + \frac{\delta}{1 - F(\theta_0) + 1 - F(\theta_1)} \right] \quad (C_0)
$$

and for all $t>1$

$$
(1 + \delta) \theta_t = \left[ \frac{1}{1 - F(\theta_{t-1}) + 1 - F(\theta_t)} + \frac{\delta}{1 - F(\theta_t) + 1 - F(\theta_{t+1})} \right] I. \quad (C_t)
$$

We now show that the sequence $\hat{\Theta}$ satisfies $\hat{\theta}_t \geq \hat{\theta}_{t+1}$. Suppose that it is not the case and define $\tau$ as the first date $t$ such that $\hat{\theta}_t < \hat{\theta}_{t+1}$. Consider the sequence $\Theta'$ such that $\theta'_t = \hat{\theta}_t$ for $t \leq \tau$, $\theta'_{t+1} = \hat{\theta}_{t+1}$, and $\theta'_t = \hat{\theta}_t$ for $t \geq \tau + 1$. By construction, this sequence satisfies $(C_t)$ for any $t \leq \tau - 1$ (the condition $(C_t)$ is then unchanged) and for $t \geq \tau + 2$ (the new condition $(C_t')$ then corresponds to the previous condition $(C_{t-1})$). Furthermore, it satisfies (for the sake of
presentation, we suppose $\tau>0$, but the reader can check that the argument applies as well to the case $\tau=0$:

$$
(1 + \delta)\theta'_t = (1 + \delta)\hat{\theta}_t \geq \left[ \frac{1}{1 - F(\hat{\theta}_{t-1}) + 1 - F(\hat{\theta}_t)} + \frac{\delta}{1 - F(\hat{\theta}_t) + 1 - F(\hat{\theta}_{t+1})} \right] I
$$

where the inequality derives from $\hat{\theta}_t < \hat{\theta}_{t+1}$, and

$$
(1 + \delta)\theta'_{t+1} = (1 + \delta)\hat{\theta}_t \geq \left[ \frac{1}{1 - F(\hat{\theta}_{t-1}) + 1 - F(\hat{\theta}_t)} + \frac{\delta}{1 - F(\hat{\theta}_t) + 1 - F(\hat{\theta}_{t+1})} \right] I
$$

whereas the inequality stems from $\hat{\theta}_{t-1} \geq \hat{\theta}_t$. It follows that the sequence $\Theta'$ is self-financing; but then, $\hat{\Theta'} = \hat{\Theta} \wedge \Theta'$ is also self-financing and satisfies $\hat{\theta}'_t \leq \hat{\theta}_t$ for any $t$ and $\hat{\theta}'_{t+1} = \hat{\theta}_t < \hat{\theta}_{t+1}$, so that $\hat{\Theta}$ was not the smallest self-financing sequence.

Next, we show that the sequence satisfies $\hat{\theta}_t > \hat{\theta}_{t+1}$. Suppose that it is not the case. Given the above argument, $\hat{\theta}_t$ must therefore remain constant over several periods. Define $\tau$ as the first date $t$ such that $\hat{\theta}_t = \hat{\theta}_{t+1}$ and $T$ as the first date $t \geq \tau$ such that $\hat{\theta}_t < \hat{\theta}_{t+1}$. Consider the sequence $\Theta'$ such that $\theta'_t = \hat{\theta}_t$ for $t \leq \tau$ and $\theta'_{t-\tau} = \hat{\theta}_{t-\tau-1}$ for $t \geq \tau+1$. As before, this sequence satisfies $(C_t)$ for any $t \leq \tau-1$ (the condition $(C_t)$ is again unchanged) and for $t \geq \tau+2$ (the new condition $(C'_{t+1})$ then corresponds to the previous condition $(C_{t+T-\tau+1})$). Furthermore, it satisfies (assuming again $\tau > 0$ for the sake of presentation):

$$
(1 + \delta)\theta'_t = (1 + \delta)\hat{\theta}_t \geq \left[ \frac{1}{1 - F(\hat{\theta}_{t-1}) + 1 - F(\hat{\theta}_t)} + \frac{\delta}{1 - F(\hat{\theta}_t) + 1 - F(\hat{\theta}_{t+1})} \right] I
$$

where the inequality derives from $\hat{\theta}_t < \hat{\theta}_{t+1}$, and

$$
(1 + \delta)\theta'_{t+1} = (1 + \delta)\hat{\theta}_t \geq \left[ \frac{1}{1 - F(\hat{\theta}_{t-1}) + 1 - F(\hat{\theta}_t)} + \frac{\delta}{1 - F(\hat{\theta}_t) + 1 - F(\hat{\theta}_{t+1})} \right] I
$$

whereas the inequality stems from $\hat{\theta}_{t-1} \geq \hat{\theta}_t$. It follows that the sequence $\Theta'$ is self-financing; but then, $\hat{\Theta'} = \hat{\Theta} \wedge \Theta'$ is also self-financing and satisfies $\hat{\theta}'_t \leq \hat{\theta}_t$ for any $t$ and $\hat{\theta}'_{t+1} = \hat{\theta}_t < \hat{\theta}_{t+1}$, so that $\hat{\Theta}$ was not the smallest self-financing sequence.
where the first inequality derives from \((C_t)\) and \(\hat{\theta}_{t+1}=\hat{\theta}_t>\hat{\theta}_t\), whereas the second inequality stems from \((C_T)\) and \(\hat{\theta}_{T-1}=\hat{\theta}_t\). Therefore, \(\Theta'\) and \(\Theta'=\hat{\Theta}\setminus\Theta'\) are both self-financing sequences; but \(\hat{\theta}_t\leq\hat{\theta}_t\) for any \(t\) and \(\hat{\theta}_{t+1}=\hat{\theta}_t<\hat{\theta}_{t+1}\), so that \(\hat{\Theta}\) was not the smallest self-financing sequence.

The smallest sequence \(\hat{\Theta}\) is thus strictly decreasing over time. Since it is bounded below by \(\theta=0\), it converges towards a value \(\hat{\theta}_\infty\) which, by continuity, must satisfy

\[
(1 + \delta)\hat{\theta}_\infty = \left[ \frac{1}{1 - F(\hat{\theta}_\infty) + 1 - F(\hat{\theta}_\infty)} + \frac{\delta}{1 - F(\hat{\theta}_\infty) + 1 - F(\hat{\theta}_\infty)} \right] \hat{I},
\]

that is,

\[
2[1 - F(\hat{\theta}_\infty)]\hat{\theta}_\infty = \hat{I}.
\]

Hence, \(\hat{\theta}_\infty=\theta^*\). □

**Appendix B. Proof of Proposition 7**

(i) The concavity of the revenue function \(\theta[1 - F(\theta)]\) ensures that the Ramsey program, too, is concave. Denoting by \(\lambda\) the Lagrange multiplier associated with the budget constraint, for \(t \geq 0\) the first-order condition is

\[
\hat{\theta}_t \frac{f(\hat{\theta}_t)}{1 - F(\hat{\theta}_t)} = 1 - \frac{1}{\lambda},
\]

and thus \(\hat{\theta}_t\) must be constant, since the left-hand side is increasing.

(ii) The budget constraint then ensures that \(\theta^R\) is the lowest root of

\[
0 = \delta\theta[1 - F(\theta)] - I + \sum_{t=0}^{\infty} \delta^t[(1 + \delta)\theta[1 - F(\theta)] - I]
= \left[ \delta + \sum_{t=0}^{\infty} \delta^t(1 + \delta) \right] \theta[1 - F(\theta)] - \sum_{t=0}^{\infty} \delta^t I
= \frac{1}{1 - \delta} \left[ 2\delta\theta[1 - F(\theta)] - I \right],
\]

or

\[
2\theta^R[1 - F(\theta^R)] = \frac{I}{\delta}.
\]

Hence \(\theta^R\) lies between \(\theta^\text{NDC}\) (the smallest root of \(2\theta[1 - F(\theta)] = I<\frac{I}{\delta}\)) and \(\theta^m\) (which maximizes \(\theta[1 - F(\theta)]\)). □

**References**


